# Theory of Trigintaduonion Emanation and Origins of $\alpha$ and $\pi$

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**Abstract**

In this paper a specific form of maximal information propagation is explored, from which the origins of $\alpha$, $\pi$, and fractal reality are revealed.

## Introduction

The new unification approach described here gives a precise derivation for the mysterious physics constant $\alpha$ (the fine-structure constant) from the mathematical physics formalism providing maximal information propagation, with $\alpha$ being the maximal perturbation amount. Furthermore, the new unification provides that the structure of the space of initial 'propagation' (with initial propagation being referred to as 'emanation') has a precise derivation, with a unit-norm perturbative limit that leads to an iterative-map-like computed $\alpha$ (a limit that is precisely related to the Feigenbaum bifurcation constant and thus fractal). The computed $\alpha$ can also, by a maximal information propagation argument, provide a derivation for the mathematical constant $\pi$.

The ideal constructs of planar geometry, and related such via complex analysis, give methods for calculation of $\pi$ to incredibly high precision (trillions of digits), thereby providing an indirect derivation of $\alpha$ to similar precision. Propagation in 10 dimensions (chiral, fermionic) and 26 dimensions (bosonic) is indicated [1-3], in agreement with string theory. Furthermore a preliminary result showing a relation between the Feigenbaum bifurcation constant and $\alpha$, consistent with the hypercomplex algebras indicated in the Emanator Theory, suggest an individual object trajectory with 36=10+26 degrees of freedom (indicative of heterotic strings). The overall (any/all chirality) propagation degrees of freedom, 78, are also in agreement with the number of generators in string gauge symmetries [4].

In 'Unified' Propagator Theory, the form of propagation is itself emergent, and within that construct, there is then emergent the functional optimization that describes how the system behaves, e.g., the Lagrangian is part of that latter emergent step. Thus, Lagrangians originally introduced as a convenient mathematical constructs, and in later physics endowed with their own physicality, especially in conjunction with the path-integral description to properly capture topological features (the Aharanov-Bohm experiments), are here seen as direct mathematical encapsulations of the fundamental emergent nature of the physical system.

The number system, or algebra, used to describe a physical system is typically the real numbers, sometimes the complex numbers (to describe wavelike phase information), and, rarely, the quaternionic numbers (to describe rotation and EM interactions). In recent theoretical efforts, attention has also been paid to octonionic numbers to describe Quantum Electrodynamics (QED) and Quantum chromodynamics (QCD) interactions [5-11]. The algebras given by real, complex, quaternionic, octonionic, sedenionic, trigintaduonionic, …., are known as the Cayley-Graves algebras, whose dimensions double at each step, one dimension for real, two for complex, four for quaternionic, etc. Maximal *unitary* propagation occurs with the octonion algebra and no higher (thus 'maximal' propagation, seemingly, only in 8 dimensions). What is actually needed in physics 'propagation' is right multiplication with a unit-norm 'propagator', for example, giving rise to a unit-norm result (then iterating to create a path from the infinitesimal propagator steps). If this is sought instead, then a chiral extension can be made from the octonions into the sedenions, and then again into the trigintaduonions, giving rise to a *maximal 'propagation', or projective emanation, in 10 dimensions* within the 32 dimensional trigintaduonions (as shown in [1-3] and summarized in the Appendix).

For Real numbers unit norm propagation is trivial, consisting of multiplying by +1 or -1. For Complex numbers unit norm propagation involves multiplication by complex numbers on the classic unit circle in the complex plane, which reduces to simple phase addition according to rotations about the center of that circle (motions on $S^1$). For quaternion numbers unit norm propagation is still straightforward since it's still, in the end, a normed division algebra, where N(xy)=N(x)N(y). For the quaternions, instead of motion on $S^1$, we now have motion on $S^3$, the unit hypersphere in four dimensions. This still holds true for Octonions, with unit norm still directly maintained when multiplying unit norm objects in general. Now the motion is that of a point on a seven dimensional hypersphere $S^7$. Sedenions are not normed division algebras, lacking linear alternativity and the moufang loop identities (see Appendix), thus multiplication of unit norm objects for sedenions (points on $S^{15}$) will not, generally, remain unit norm, i.e., will leave the $S^{15}$ space.

The question then arises is there is a sub-algebra or projection in the sedenions, that is not just trivially the octonions, that can still allow unit norm propagation? If this works for Sedenions, what about Bi-sedenions and higher dimensional Cayley algebras? In [1] and the Appendix it is shown that there are two Sedenion subspaces where the unit norm property is retained. This is found again at the level of the Bi-Sedenions by a similar construction. The results were initially explored computationally [1], then later established in theoretical proofs [1-3]. In those proofs a key step fails when attempting to go to higher orders beyond the bi-sedenions and its sub-algebra propagation. (Propagation is taken to mean that a unit norm element of an algebra when multiplied by a unit norm element that can be propagated: (unit norm)*(unit norm subalgebra)=(unit norm), where the one-sided multiplication by the special subalgebra results in a product that remains unit norm.)

In the RCHO(ST) hypothesis Physics unification was thought to directly entail propagation in terms of hypercomplex numbers [12] (from Reals thru Trigintaduonions in Cayley sequence), This hypothesis was motivated by Maxwell, Feynman and Cayley, in hopes of being able to directly encode the standard model and statistical mechanics. But to get the 10D propagation formalism entails 'projections', not the more familiar mathematical objects directly giving rise to standard propagation (in a complex Hilbert space). Instead, the standard propagation is part of the emergent (with complex Hilbert space) description, as will described further in later sections.

The Feynman-Cayley Path Integral proposed in [1,2] involved use of chiral bi-sedenions in an effort to identify a mathematical framework within which to have a unified propagator theory (and maximal information propagation was sought for such a hypothesized propagator). At its root, this is a hypothesis for an algebraic reality, with algebraic elements describing 'reality' and algebraic multiplicative processes underlying propagation. All of the different 'paths' of propagation are then brought together in a sum – where stationary phase is selected out and the variational calculus basis for much of physics then takes over to offer all of the familiar elegant solutions of classical physics. This is still thought to be the process, but two stages of emergence are indicated: (1) emergence of the emanation (projective) propagations followed by the (2) emergence of standard propagation in a complex Hilbert space. So, even though we start with RCHO(ST) with the emergence of *emanation*, we end with a framework for emergence of standard propagation where that propagation involves a complex Hilbert space in order to 'propagate', and not any of the other algebras involved in RCHO(ST).

So, if the Feynman-Cayley construction works on all algebras, it essentially allows a selection argument to be made for the highest order unit norm propagating algebra in devising theories to describe matter. The highest order propagating structure might, thus, be the ten dimensional (10D) unit-norm bi-sedenion elements, that are shown here, that are (chirally) extended sedenions that are themselves made from chirally extended octonions. The nine dimensional space "free" dimensionality when paired with the implicit time dimension provides a 10 dim (1,9) spacetime theory, in agreement with string theory. (If the time is augmented to be complex, then we get an 11-dim theory, with a fundamental role for Euclideanization related thermodynamics properties.)

In the Results we begin with constructing the theoretical expression for a general element of the bi-sedenion algebra after two chiral bi-sedenion multiplicative propagation steps. A simple analysis of the number of terms in this expression, when reduced to three-element algebraic 'braid-level', results in a count on algebraic braids of 137, plus a little extra (e.g. some lagniappe for the best 'cooking') of a contribution towards a 138$^{th}$ braid. (The extra involves a complex-dimensional extension outside the 10-dim propagation). Once this is done, the Results then show derivations for $\pi$ and the Feignebaum bifurcation constant. To put the Results in their proper context, it will help to review some of the background and mystery associated with the famous parameters discussed, starting with the fine structure constant.

## Background

### The mystery of alpha

The fine-structure constant, $\alpha$, has been a mystery confounding physicists for over a century. In early work on spectral analysis where it first appeared, Sommerfeld noted the almost cabbalistic underpinnings of the mathematics (in his book Atombau and Spektrallinien [13], Sommerfeld referred to the Rydberg top square equation as a 'cabbalistic' formula). Wolfgang Pauli, a student of Sommerfeld's, shared his keen interest in the origins of $\alpha$ and turned it into a life-long obsession. So much so, that it practically drove him mad, to where he sought the help of famed psychoanalyst Carl Jung, with whom he eventually partnered to try to solve the mystery of $\alpha$ (the madness is contagious). From Pauli's Nobel Prize Lecture:

> "From the view of logic my report on 'Exclusion principle and quantum mechanics' has no conclusion. I believe it will only be possible to write the conclusion if *a theory will be established which will determine the value of the fine structure constant* and will thus explain the atomistic of electric fields actually occurring in nature." (emphasis mine)

The obsession with $\alpha$ continued with the next generation of great Physicists as well, particularly Feynman, who said [14]:

"There is a most profound and beautiful question associated with the observed coupling constant, *e* – the amplitude for a real electron to emit or absorb a real photon. It is a simple number that has been experimentally determined to be close to 0.08542455. (My physicist friends won't recognize this number, because they like to remember it as the inverse of its square: about 137.03597 with about an uncertainty of about 2 in the last decimal place. It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it.) Immediately you would like to know where this number for a coupling comes from: is it related to pi or perhaps to the base of natural logarithms? Nobody knows. It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the "hand of God" wrote that number, and "we don't know how He pushed his pencil." We know what kind of a dance to do experimentally to measure this number very accurately, but we don't know what kind of dance to do on the computer to make this number come out, without putting it in secretly!"

**Fractal Reality**

Consider maximum "unit-norm" propagation (via right multiplications), e.g., a projection (or 'emanation'), where a hypercomplex 'emanator' has maximum propagation dimensionality ten, residing in a double-chiral 10dim subspace of the 32dim space of trigintaduonions. The maximum propagation perturbation allowed from the 10dim space into the embedded 32 dim space is given by the fraction $\alpha$ for the non-10dim part, where this is taken as the definition of $\alpha$. Computational efforts to determine $\alpha$ recover the known $\alpha$ from QED, as in [2].

Exploration to high precision indicates a possible fractal limit (as noted in [2]), with possible pattern recurrences as in the Mandebroit Set on complex numbers (there a limit on complex numbers, here a limit on hypercomplex numbers, the trigintaduonions, operating in 32x2dim, x2 from complexation, instead of 2dim). A further complication is that the 32 dim hypercomplex trigintaduonion numbers have also become non-associative (but still retain octonionic sub-space 'braid' rules, which are critical in what follows).

To see the fractal connection, consider the iterative mapping based on the function $z_n = (z_{n-1})^2 + c$. For choice c and initial $z_0=0$, if $z_\infty \rightarrow \infty$, then that c is outside the set, otherwise, if remains bounded, then it's in the (Mandelbroit) set. This is an example famous for its beautiful fractal images and mathematical properties. The largest c value (at the edge of chaos) is known as the bifurcation parameter and is $c^* = 1.401155189\ldots$. The maximum allowable 'perturbation' for z (not $z^2$) would then be $(c^*)^{(1/2)}$. In the trigintaduonion propagation we discover in what follows we have chiral propagation in the 32 dim trigintaduonion space, where the real dimension is fixed by the unit-norm property, leaving 31 'free' imaginary dimension/parameters, two of which are selected for the chirality (and thereby fixed in their real component). If we allow the same maximal bifurcation parameter as a factor for each of the 29 free dimensions (and for the imaginary part in the other 3 dimensions), and allow maximal antiphase on the imaginary parts of the 2 chirally-fixed dimensions (a relation obtained in the more careful derivation to follow), we can estimate the max perturbation allowed for the trigintaduonion emanation to be

$$\text{Estimated } \alpha^{-1} = (c^*)^{(1/2)(29+\pi/36+\pi/36)} = (c^*)^{29/2+\pi/36} = (1.401155189\ldots)^{29/2+\pi/36} = 137.035\ldots.!$$

A more precise relation is given in the Results.

**The emergence of Propagation: Part I**

Consider an infinite-order hypercomplex unit-norm Number giving rise to a propagating structure, with time and chirality selected, with QED and QCD gauge bundles emergent and their associated 22 parameters fixed ($\alpha$ is fixed too). With the emergence have the 10dim propagation derived in what follows, with $\alpha$-perturbation into the full 32 dim. (The higher order Cayley-Graves algebras may have Planck-scale perturbations, but have zero-divisors acting as effective cutoffs.) Thus, have a hypercomplex Big Bang, and an 'emergence' where a construct for 'receiving' that emergence is found (the unit norm base state/word and the random propagation step that moves forward the universe on step at a time….). The receiving of the universal emanation results in emergent spacetime and chirality, perhaps akin to the emergence of Amman bars and orientation with a Penrose tiling once seeded [15]. Random addition from initial seed of Penrose tiling versus random initial norm1 state with random unit-norm 10 dim chiral propagations (within a 32dim space), appears to have emergent structure as well.

Once a 10dim propagation is emergent, there is likely an emergent semiclassical string theory. The emergence process also helps explain the validity of the various renormalization methods (dimensional regularization, in particular). In the latter regard, the dimensional regularization trick whereby a higher complex dimensional extension is invoked is here seen to actually be true. Similarly, string theory is an emergent construct, along with the manifold and the standard model, and Lagrangian encapsulations, etc. Thus, invoking a higher dimensional space, often through complexification of real variables, is natural in this emergent context, where such a higher dimensional complex embedding is already posited to exist in the emanation emergence process. The complex-extension method is critical in QED, Euclideanized path integral formulations, and thermal quantum field theory in general, where complex time relates to introducing a thermal background temperature for the system (thus the complex extension allows unification with thermal physics and emergent, Law of Large Numbers (LLN) based, statistical mechanics constructs).

Consistency with the semiclassical first quantized string theory, allowing an alternative renormalization, also indicates the flat-space oddity of the seemingly general formalism of string theory (in other regards) having an odd flat spacetime reference. This is here understood as simple consistency with the maximum information propagation in the universal algebra formalism, where the 10 dimensions are resulting from the 'free' algebra parameters in the 32 D trigintaduonions, and as such have no other structure between them other than the implied 'flat metric' of the trigintaduonion algebra. This also demotes the string to being an artifact of the emergence, albeit on a higher level than the quantum field theory based on point particles descriptions usually sufficient.

In [6], with split octonions alone it is possible to describe spacetime, EM-fields, and uncertainty relations…

This is very promising as regards extracting the familiar standard model from the much larger, already chiral, 10D propagation (fermionic) with maximal perturbation $\alpha$ (and 22 parameters from the non-propagating dimensionalities [3]). From this we get an achiral (4-chiral sum) subspace with 26Dim propagation for bosons. Get complete propagation with 78 generators (consistent with string theory, as is the 10dim and 26 dim). Also, we shall see that we have 137 tri-octonionic 'braids' of information flowing in the 10dim chiral propagation, this is critical in the derivation of $\pi$ from $\alpha$ that follows.

**Results**

**Trigintaduonion Emanation and Emergence of the Critical Parameters 78, 137, and $\alpha$**

Consider a general Norm=1 (32D) Trigintaduonion (Bi-Sedenion): (A,B), where A and B are sedenions (16D).
Then have (A,B) = ( (a,b), (c,d) ), where {a,b,c,d} are octonions.

Slightly different than a propagator, we have an 'emanator' with the following notation and properties: Emanator describes a 10D multiplicative step. The emanator is a chiral bi-sedenion: a trigintaduonion whose first sedenion half is itself a chiral bi-octonion, and the second sedenion half is a pure real (as is the second octonion half): $(\tilde{A},\beta)$, $\tilde{A} = (\tilde{a},\alpha)$, where the norm is 1, $\alpha$ is a real octonion, and $\beta$ is a real sedenion. Thus:

Emanator: $(\tilde{A},\beta) = ( (\tilde{a},\alpha), \beta)$.

Note: $\tilde{A}^* = (\tilde{a}^*,-\alpha)$.

Let's set up a description of the Universal 'Emanation' resulting from a few emanation steps. To begin, suppose we have already arrived at, or received, a unit norm trigintaduonion (32D) state 'T', and suppose our emanations are the result of right multiplication with a chiral bi-sedenion 'step', and suppose we consider one such path after just a few steps. Here's the notation to begin:

$\mathbf{T}$ = (A,B), a unit norm trigintaduonion.
$\boldsymbol{\tau} = (\tilde{A},\beta) = ( (\tilde{a},\alpha), \beta)$, the 'emanator' above (so named to distinguish from a 'propagator').

Universal Emanation from T on single path with three steps:

$( (\mathbf{T} \bullet \boldsymbol{\tau}_1) \bullet \boldsymbol{\tau}_2) \bullet \boldsymbol{\tau}_3) \ldots$

Consider the first emanation step:

$\mathbf{T} \bullet \boldsymbol{\tau}_1 = (A,B) \bullet (\tilde{A},\beta) = ( [A\bullet\tilde{A}-\beta^*\bullet B] , [B\bullet\tilde{A}^*+\beta\bullet A] )$. (Standard Cayley algebra multiplication rules.)

$A\bullet\tilde{A} = (a,b) \bullet (\tilde{a},\alpha) = ( [a\bullet\tilde{a}-\alpha^*\bullet b] , [b\bullet\tilde{a}^*+\alpha\bullet a] )$

$B\bullet\tilde{A}^* = (c,d) \bullet (\tilde{a}^*,-\alpha) = ( [c\bullet\tilde{a}^*+\alpha^*\bullet d] , [d\bullet\tilde{a}-\alpha\bullet c] )$

Thus,
$\mathbf{T} \bullet \boldsymbol{\tau}_1 = (A,B) \bullet (\tilde{A},\beta) = ( [ (a\bullet\tilde{a}-\alpha^*\bullet b-\beta c) , (b\bullet\tilde{a}^*+\alpha\bullet a-\beta d) ] , [ (c\bullet\tilde{a}^*+\alpha^*\bullet d+\beta a) , (d\bullet\tilde{a}-\alpha\bullet c+\beta b) ] )$.

At the lowest octonion level, that covers the pure real trigintaduonion, we have:

$(a\bullet\tilde{a}-\alpha^*\bullet b-\beta c) \rightarrow 8x8 + 8 + 8 - 2 = 64+14 = 78$ independent octonion terms (78 independent generators of motion). The −2 comes from the unit norm constraints on T and τ.

Now consider the second propagation step:

$(\mathbf{T} \bullet \boldsymbol{\tau_1}) \bullet \boldsymbol{\tau_2}) = ( [ (a\bullet\tilde{a}-\alpha^*\bullet b-\beta c) , (b\bullet\tilde{a}^*+\alpha\bullet a-\beta d) ] , [ (c\bullet\tilde{a}^*+\alpha^*\bullet d+\beta a) , (d\bullet\tilde{a}-\alpha\bullet c+\beta b) ] ) \bullet (\tilde{A},\beta)$,

where $\boldsymbol{\tau_2} = (\tilde{A}',\beta') = ( (\tilde{a}',\alpha'), \beta')$.

Let $(\mathbf{T} \bullet \boldsymbol{\tau_1}) \bullet \boldsymbol{\tau_2}) = ( [Z_{11},Z_{12}] , [Z_{21}, Z_{22}] )$.

$Z_{11} = (a\bullet\tilde{a}-b\alpha-c\beta)\bullet\tilde{a}' - (b\bullet\tilde{a}^*+\alpha a-\beta d)\,\alpha' - (c\bullet\tilde{a}^*+d\alpha+a\beta)\beta'$.

In $Z_{11}$ we can replace the octonions with their unit component forms:

$a = a_1e_1 + a_2e_2 + \ldots + a_8e_8$ ,

where $\{e_1, e_2, \ldots, e_8\}$ are the unit octonions (one real, seven imaginary), while '$\alpha$'$=\alpha e_9$ and '$\beta$'$=\beta e_{17}$, originally, but in expressions, are reduced to just their real part. All expressions, thus, involve 10 components: $\{e_1, e_2, \ldots, e_8, e_9, e_{17}\}$, and as the equations for $Z_{11}$ shows, grouped in factors of three (three-element octonionic 'braids'). We don't have associativity but we do have alternativity and the braid rules on three-element octonionic products that allows their regrouping. Applying these rules to have only ordered $e_i\bullet e_j\bullet e_k$ products in a simplified expression, we will then have $10x9x8/3! = 120$ independent terms when the products involve different components. We have 8 independent terms when the first product are on the same component (equals 1), have 8 independent terms when the second product involves the same component, and have 1 independent term when the three-way product equals 1. There are, thus, 137 independent terms in $Z_{11}$, where each term has norm less than unity (since each octonionic component has norm less than one and the norm of a product of octonions is the product of their norms). The terms involving products with the same component, or with the components three-way product equal unity, are correspond to the 'telescoping terms' in what follows.

When $\mathbf{T=((a,b),(c,d))} \rightarrow ((\mathbf{T} \bullet \boldsymbol{\tau_1}) \bullet \boldsymbol{\tau_2})=((Z_{11},Z_{12}),(Z_{21}, Z_{22}))$. we have $a\rightarrow Z_{11}$ and the terms involving 'a' in $Z_{11}$ are referred to as 'telescoping' due to their simple math properties with further emanation steps. In particular, the terms involving 'a' are:

$Z_{11}[a\text{ terms}]= a\bullet\tilde{a}\bullet\tilde{a}' - a\alpha\alpha' - a\beta\beta'$.

We can see that the original 'a' information is passed along three (telescoping) channels, one involving repeated full octonionic factors $\tilde{a}$, one involving repeated real-octonion $\alpha$ factors, and one involving repeated real-octonion $\beta$ factors:

(1) $a \rightarrow (a\bullet\tilde{a})\bullet\tilde{a}'$ , if this product is continued indefinitely, then we have the random product of a collection of octonions, all of which have norm less than one (although their norms can be quite close to one). If their norms were perfectly equal to one, then the addition of their random 'phases' would tend to

cancel to zero, giving only a real octonionic component (same argument for phase cancelation on S1 as on S7 or S15). What results is a 'mostly' real octonion, having some imaginary part. Note, we have reached a situation where we know there is unit-norm propagation, but where some bounded imaginary components can arise. Regardless, we have one independent component, and one, *dependent*, imaginary, component (so not counted)

(2) a → $a\alpha\alpha'$ , if this product is continued indefinitely, 'telescoped' with repeated $\alpha$ products, we see that the original 8 independent terms arising from 'a' are passed forward with an overall real octonion product, giving rise to 8 independent terms.

(3) a → $a\beta\beta'$ , as with (2), we have 8 independent terms.

From the above, we see an alternative accounting of the extra 17 independent terms to go with the 120 for a total of 137 independent terms in the propagation of the real octonionic sector of the universal propagations (the other octonionic sectors in the bi-sedenion emanation are surmised to have a similar 137-term transmission channel by similar accounting). A benefit of the telescoping analysis is it clarifies how in (1) an imaginary component may arise, and in perturbation expansions it will then be natural to refer to an overall imaginary component.

There are 137 terms in the dually chiral 'emanation', each with norm bounded by unity, with total bi-sedenion norm equal to unity. In the analysis that led to the computational discovery of $\alpha$ [1,2], an imaginary (non 10D) component was added of growing magnitude until unit-norm propagation failed. In essence, a maximum perturbation, from propagation strictly in the 10D subspace of the 32D trigintaduonions, was sought. Now, once we purposefully allow imaginary perturbative contributions in the definition of the 'emanator', then all is the same in the above analysis, except now the imaginary component in (1) is *independent*, albeit bounded to significantly less magnitude than unity (we will find it is 137.035999-137=0.035999).

In the next section we identify maximal perturbation by doing an independent 'braid' term analysis, and by adding a maximum perturbation term that implicitly identifies a definition of maximum antiphase. From this definition of maximum antiphase, there results the parameter $\pi$, but here with an unexpected origin from emanation theory where maximum perturbation occurs at maximal antiphase.

**The emergence of $\pi$ from $\alpha$**

In the relation of Gilson [16], to be derived and explained here, the key parameters are those indicated in the chiral subspace in the maximum perturbation definition for $\alpha$. Suppose we take $\alpha$ as a given, with maximal value, by consistency, related to subspace structures of the trigintaduonions, can we express a relation to the classic mathematical parameter $\pi$? The answer is yes, and that's the derivation that follows.

In what follows we will use the previously identified property that there are 137 independent tri-octonionic braid propagations contributing to the $\alpha$ max-perturbation 10-dim propagation. When perturbations are allowed, where each braid has a small contribution in each of the trigintaduonion's 32 dimensions, minus the unit-norm constraint and two choices of chirality constraints eliminating 3 of those dimensions, leaving 29 dimensions free. The 137 independent (real) tri-octonionic braids each contribute a maximum magnitude 1 to the propagation (since they can have at most the entire unit-norm flow at any given moment), with total real magnitude of the braids being '137'. Now consider that there is an overall complex octonionic contribution, representing the perturbation contribution in its entirety (as alluded to in the previous section), whose magnitude is selected such that the overall magnitude is that providing the complex magnitude at maximal antiphase contribution to the real magnitude of 137. Let's denote the first maxima of complex magnitude antiphase encountered, as phase is increased from zero, as being at phase = '$\pi$'. In terms of the magnitudes we thus have the magnitude relation in Fig. 1:

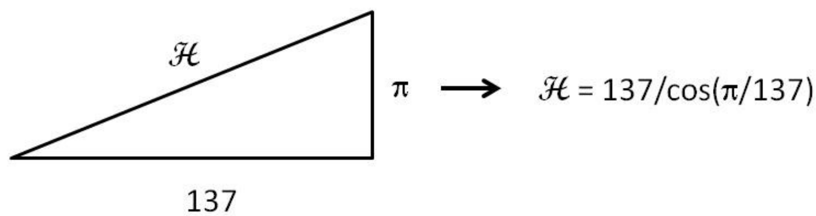


Fig. 1 The magnitude relation for effective braid count. First estimate based on magnitudes.

We want $\alpha\mathcal{H} \leq 1$ at all times in order to not break the unit-norm property, where $\alpha$ is the maximum perturbation allowed. Thus we have $\alpha_{max} = 1/\mathcal{H}$ for our initial estimate.

As mentioned previously, for the propagation of any braid, given the unit norm and chiral-selection constraints, there are 29 possible dimensions of propagation, thus 29 imaginary components to consider in addition to the real contribution on that thread. Now imagine that the $\pi$ (maximal antiphase amount) for maximal perturbative information flow (still unit norm) occurs with that antiphase contribution equally distributed amongst all 137 braids and their 29 dimensional propagations, thus have an imaginary contribution that we can calculate from the given overall magnitude, but now with respect to a phase angle of $\pi/(137 \times 29)$ as shown in Fig. 2:

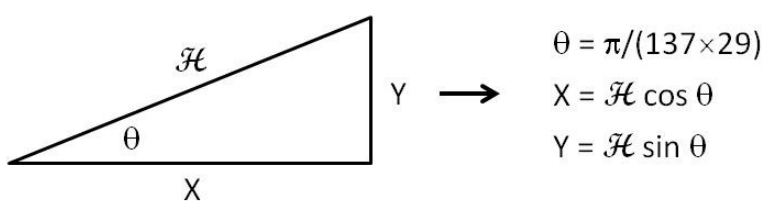


Fig. 2. The magnitude-angle relation on effective transmission pathways (all with norm bounded by 1.0), where angle is for $\pi$ in 137x29 subparts.

Since the magnitude of our braid contributions must not exceed unity, lets rescale by $\mathcal{H}$, to get $Y = \sin(\theta)$. Consider what we have now, a description relating the relative amounts of the real and imaginary parts of the chiral propagation for each of the braid propagations into the 29 'free' dimensions available. The imaginary part is now $\sin(\pi/(137\times29))$ for each of these contributions. If we were to arrive at a perfect antiphase arrangement of all $(137\times29)$ of these components we'd have a total antiphase of $(137\times29)\sin(\pi/(137\times29))$, which isn't simply '$\pi$' as desired. So lets consider a last triangle manipulation, where now a simple rescaling is done from $\sin(\pi/(137\times29))$ to $(\pi/(137\times29))$, then a rescaling of the total braid magnitude (Fig 3):

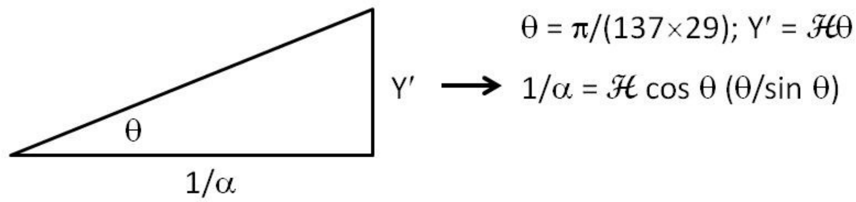


Fig. 3. The magnitude-angle-side relation on effective transmission pathways (all with norm bounded by 1.0), where angle is for $\pi$ in 137x29 subparts and imaginary side length is rescaled such that.it is $\pi/(137\times 29)$, such that maximal antiphase from 137x29 contributions is $\pi$.

Thus, the maximum value of the real component of the propagation defines a limit on perturbation amount where $(1/\alpha_{max}) = \mathcal{H}\cos\theta\,(\theta/\sin\theta)$. We thus find a unique relation between $\alpha$ and $\pi$:

$$\alpha_{max} = (1/137)(\cos\beta/\cos\theta)(\sin\theta/\theta), \text{ where } \beta = (\pi/137) \text{ and } \theta = \pi/(137 \times 29)$$

$$\alpha_{max} \cong (1/137)(0.9997370885/0.9999996874)(0.000790735547/0.000790735629)$$

thus,

$$1/\alpha_{max} \cong 137.0359998,$$

where the last digit is uncertain given the precision used.

Since $\alpha$ is a fundamental parameter that emerges for a maximal propagation, and we find here another relation on $\alpha$ that ties it to the maximal antiphase amount '$\pi$', we find that this is the origin of the fundamental parameter $\pi$ from mathematics. Although the idealizations of planar geometry can be used to derive $\pi$ (or modern variants from complex analysis involving the complex plane) it is interesting that we have here an origin of $\pi$ via what leads to maximal anti-phase when computing $\alpha_{max}$ , where $\alpha=\alpha_{max}$ is selected for maximal information propagation. So $\pi$ 'came first' (with the notion of antiphase) then the induced idealizations of planar geometry. In split octonionic representations $O=(Q_L,Q_U)$, the lower quaternion $Q_L$, can separate as a set of spacetime coordinates obeying Lorentzian transformations, and whose spatial part has the 3-dimensional continuation of the implied planar geometry constructs (now identified as emergent via $\pi$), thus we have emergence of familiar geometry (and Lorentzian transforms are natural from the quaternionic sub-algebra).

**The relation of $\delta$ to $\alpha$ (and thus $\pi$)**

The relation of $\alpha$ to the Feigenbaum bifurcation parameter $\delta$, involves seeing the limit on maximum $\alpha$ computation as related to the maximum stable value ($\delta$) on the one (complex) parameter iterated mapping (but now involving roughly 32 parameters for the 32D trigintaduonion). A more precise relation can now be given, using arguments similar to those employed in the previous section in showing the relation between $\alpha$ and $\pi$, and we have:

$$\text{Theoretical } \alpha^{-1} = (c^*)^{\gamma} = 137.035999\ldots,$$

where,

$$\gamma = (1/2)(29 + (\pi/36)[\ 1/(29x137x28) + 1/(29x137x2) + 2\ ]).$$

A preliminary description of where each term in the above expression comes from is as follows. Dimensional analysis on the iterative mapping has c* ≈ (perturbation squared), thus $(c^*)^{(1/2)}$ for the amount of perturbation per dimension. There are 29 "free" dimensions for the chiral trigintaduonion perturbations, since one is dependent via the norm=1 relation and two are dependent via the real octonion and real sedenion constraints defining the two chiralities (e.g., 32-1-2=29). Thus, 29 factors of the square root of the Feigenbaum bifurcation parameter. Now consider the add-ons for each of the dimensionalites in their contribution via imaginary components. A preliminary theoretical form might have one term, maximal antiphase, associated with an overall imaginary contribution to the real 29 dimensions, and two terms, maximal antiphase, associated with the real octonion and real sedenion each having an overall imaginary contribution at maximal antiphase, likewise the overall unit norm seen as real component with imaginary contribution also increased to maximal antiphase. All of the antiphase terms have a factor ($\pi$/36) consistent with the (maximal) $\pi$ phase partitioned according to the degrees of freedom of the information transmission object (having 36 dimensions, consistent with 36=10+26 in the heterotic string). For the two chiral transmissions, this completes the partitioning, thus the '2'. For the overall imaginary term at maximal antiphase associated with the 29 'free' dimensions, there are forward threads of propagations according to each dimension (29), the number of independent tri-octionic transmission 'braids' (137), and the number of degrees of freedom of the overall flow according to 78 generators of motion, minus 22 fixed parameters, halved for forward propagations only ((78-22)/2)=28), thus have the term '1/(29x137x28)'. For the overall imaginary with maximal antiphase associated with the real component of the trigintaduonion have flow into 29 dimensions, on 137 threads each, and with both forward moving in time and backward, thus a factor of two, to give rise to the factor '1/(29x137x2)'.

Thus, there appears to be a relation between the Feigenbaum (or Mandelbroit) bifurcation parameter and $\alpha$ and $\pi$. Furthermore, the suspected fractal limit behavior of the (maximal) $\alpha$ computation (described in [2]) appears to be confirmed by this relation. We already saw that $\pi$ is derivative from $\alpha$, and in the relation to the bifurcation parameter, reduces to an expression that can be made with just $\alpha$ (or just $\pi$ -- where the relation to just $\pi$ proves irrationality and transcendental for $\alpha$ and the Feigenbaum constants). The choice of 36 in the relation also has special significance, as it indicates 36 degrees of freedom for the propagation an individual fundamental object and this is consistent with heterotic string theory, where the strings have 10 degrees of freedom for fermionic plus 26 degrees of freedom for bosonic, for a total of 36 degrees of freedom [4].

## Discussion

### The emergent trigintaduonion universal algebra and chiral emanation hypothesis

Physics has a lengthy 'love-hate' relationship with hypercomplex numbers. One of the earliest formulations of electromagnetism by Maxwell involved quaternionic mathematics, and even at that time this relationship was off to a difficult start. As stated by Maxwell in a manuscript on the application to electromagnetism in November of 1870 [17]: "... The invention of the Calculus of Quaternions by Hamilton is a step towards the knowledge of quantities related to space which can only be compared for its importance with the invention of triple coordinates by Descartes. The limited use which has up to the present time been made of Quaternions *must be attributed partly to the repugnance of most mature minds to new methods involving the expenditure of thought* ..." (with emphasis mine). The enthusiasm of

Maxwell for use of Quaternionic mathematics did not win over the great physicists of his day, Josiah Willard Gibbs and Oliver Heaviside in particular, who discarded the quaternionic mathematics in favor of a new mathematics (vector calculus) that they invented so as to avoid the 'foreign' hypercomplex mathematics. In a biography of Hamilton [18], in a quotation attributed to Gibbs: "My first acquaintance with quaternions was in reading Maxwell's E.&M. where Quaternion notations are considerably used. ... I saw, that although the methods were called quaternionic the idea of the quaternion was quite foreign to the subject."

The stigma associated with hypercomplex mathematics, and the higher-dimensional physics unification attempts of Maxwell and later Einstein, was still significant decades later when Feynman obtained an unusual proof of the homogeneous Maxwell equations [19-22] in a higher (than 3) dimensional space. Feynman was trying to see if any new theoretical theory would be indicated and the fact that he had obtained a novel new way to explain the existing Maxwell's equations in higher dimensions was not interesting at the time. The inextricable problems of quantum gravity and the discovery of higher-dimensional string theory, among other things, have changed the focus since that time almost 70 years ago.

It has been shown in numerous papers that the (1,9) dimensional superstring has a natural parameterization in terms of octonions [23-25]. In [5,6] the Dirac and Maxwell equations (in vacuum) are derived using octonionic algebras. In [7] a quaternionic equation is described for electromagnetic fields in inhomogeneous media. In [8], the D4-D5-E6 model that includes the Standard Model plus Gravity is constructed using octonionic fermion creators and annihilators. In [9] octonionic constructions are shown to be consistent with the $SU(3)_C$ gauge symmetry of QCD. It would appear that there are a number of implementations involving hypercomplex numbers that are consistent with the Standard Model. But there is still the question of why bother? What is shown here is why the bother might be worth it as a critical new link to string theory is provided, that may explain what dimensional compactification will relate to what experiments involving the standard model, and the formalism also allows for an explanation for Dark matter, all in a mathematics that can be absorbed into a Lagrangian formulation that could be consistent with a theory of Gravity.

To be more specific as regards the different strings. Type I superstring theory is an "open" string theory with critical dimension 10, with strings unoriented, and gauge SO(32). For closed string theories the left and right moving modes no longer have to be of the same type. If they are the same and don't obey supersymmetry, then D=26 and there are tachyons. If they are the same and obey supersymmetry (Type II), then the critical dimension is 10 with no gauge but two supersymmetries. If they are a mix with D10 'right-movers' and D26 'left-movers' (with 36 degrees of freedom), they are known as 'heterotic'. For heterotic strings with the two critical dimensionalities (D=10 and D=26), the 26 must compactify 16 as gauge degrees of freedom to reduce to 10. If compactification done with gauge E8xE8, spin(32)/Z2, or SO(16)xSO(16), then anomaly and tachyon free. Note, from [4]: "E6 is a subgroup of E8. E6 has 78 generators that form a sub-algebra of E8. E6 has a maximal subgroup SU(3)xSU(3)xSU(3)."

Often overlooked, but critical to the 8+2 emanator hypothesis, is that a relation between spinors and vectors is required in classical superstring theory, and this can only happen when the space of direction perpendicular to the string worldsheet forms a normed division algebra [26,27]. So, classical superstring theories must exist in 8+2=10 dimensions as well.

**Unit-norm propagation**
For physical description a unit norm object can be used to represent a system, and by repeated transformation to other unit norm objects, it thereby evolves. Mathematical objects that can effect this 'transformation' simply by the rule of multiplication would be objects like division algebras, ideals, and what I'll simply call projections or emanations. In the universal propagator we have a unit norm trigintaduonion (32D) and perform a right multiplication with a chiral (10D) unit norm 'alpha-step' (defined by a max perturbation $\alpha$ into the 29 free dimensions given by 32 minus one for each chiral choice, and one for the unit normalization overall). Consider multiplication of a given (starting) trigintaduonion from the right with a chiral bi-sedenion as a 'projection' through the (chiral) step indicated. The repeated application and repeated 'chiral steps' thereby arriving at a path describing a chiral propagation. The resulting universal propagation consists of a 32D unit norm trigintaduonion with propagation via right multiplication using a unit-norm, chiral bi-sedenion, with max-$\alpha$ perturbation. Thus, we suggest a projection (or emanation) from an infinite space to an infinite-order Cayley-Graves algebraic space to a 32 dim trigintaduonion space to a 10 dim chiral 'propagation' space, where the parameter $\alpha$ arises in the limit of maximum information propagation, as does the familiar mathematical constant $\pi$.

We thereby arrive at a 'Universe Propagator' that takes on the parameters desired and imprints them onto the 10D (ten dimensional) evolution as seen from the 'internal reference frame' where we reference an object in the 4D spacetime with 6D gauge field, and where the standard Lagrangian emerges as the necessary 'propagate-able' structure (Hilbert space must be complex, not real, quaternionic or octonionic, etc. [28]). From maximum information flow with the constructs, and the required emergent complex Hilbert space (thus complex path integral, thus standard quantum operator formalism) we arrive back at the familiar results with justification of their core mathematical representations (e.g., complex Hilbert space), and now with justification of all parameters, all from the emanation hypothesis. This can also be said to allow for an emergent superstring representation, with lots of additional consistencies with the Universal propagator, at the level of generators (78) for example.

Just from the propagation structure on one path we have already seen core emergent structure that results in a universal emanation with structural parameters 10,22,78,137 and perturbation maximum $\alpha = \sim 1/137$. The central notion in the universal emanation hypothesis is that there should be maximal information flow, where this is accomplished by finding the highest theoretical dimensionality of unit-norm 'propagation', here called an emanation, which turns out to be 10, then add the maximal perturbation that still allows unit-norm propagation, where that perturbation is into whatever space the 10D motion is embedded in, here a 32 dimensional (trigintaduonion algebra) space.

Given maximum information flow, the universal emergence will arrive at the 10D propagation splitting (compaction) into spacetime geometry and matter gauge fields. The parameters and structure described are consistent with string theory and quantum field theory, where we fundamentally arrive at emergence of 'propagation' as conventionally known, with a complex Hilbert Space (not other hypercomplex). A complex Hilbert Space description is the only one with propagation [28], thus it is necessarily the emergent construct that must encapsulate the geometry/matter split/compaction, into the familiar Standard Model formulations. This ties into emergence of the standard formalisms of QED and QCD. Likewise for the emergence of elegant geometrically optimal solutions relating to General Relativity (GR). Where there was conflict between QED/QCD and GR, e.g. the question of Quantum Gravity

(QG), it will be solved by considering the universal emanation of not just one path but all paths, summed with the usual phase cancelations down to a 'classical path' with stationary phase. The latter, in this context, is the emergence of standard propagator theory with standard model. So proposing here an earlier phase of universal evolution described by a theory of emanations, where mathematically invariant emergent structures appear. From this early phase, one of the emergent constructs is the familiar path integral based on standard (unitary) propagators in a complex Hilbert space.

The implication of an emergent phase of universal evolution with standard propagators, etc., is not only a framework within which to answer the questions of quantum gravity, but also a framework where the emergent trajectory has emergent 'time' (and parameters $\hbar$ and $k_B$, and euclideanization/thermality). In the end, the Black Hole (BH) conundrum in QG might reduce to a scattering calculation, where semiclassical string theory (at 1$^{st}$ quantization as known) may suffice, once 'boundary terms' are understood. With reference to the originating 'emanator' construct, we have a higher level 2$^{nd}$ Q but not based on standard propagators, but emanators. Full 2$^{nd}$ Q might shift to a notation where the stringiness is no longer discernable, and the trigintaduonion (bi-sedenion) structure dominates.

To recap: $\alpha$, 10,22,78,137, are parameters resulting from analysis on a single path construct, where the number 22 corresponds to the number of emergent parameters in the description of the propagating construct. In addition, the time choice is emergent via a multi-path construct, along with the propagator construct, and is coupled in both time step (by $\hbar$) and imaginary time increment ( $k_B$ and with Euclideanization regularization 'built in'). The formulation is inherently embedded in a higher dimensional complex space, thus all of the QFT complex analysis analyticity tricks are valid as the assumptions made are now part of the maximal information flow emergent construct.

**Maximal Information Propagation requires a complex Hilbert Space [28]**
As mentioned previously, according to [28], a complex Hilbert space is selected by the quantum deFinetti theorem, since it is required for information propagation (and thereby a restatement of the maximum information propagation concept). Because it's a complex Hilbert space, this explains why the path integral operates in a complex space, even though the underlying universal algebraic construct from which it is emergent is hypercomplex.

From Caves [28], where a quantum deFinetti Theorem requires amplitudes to be complex. Suppose f(n) is the number of real parameters to specify an n-dimensional mixed state. For real amplitudes f(n)=n(n+1)/2, for complex amplitudes f(n)=n^2, and for quaternionic f(n) = n(2n-1). For propagation, etc., need f(n1n2)=f(n1)f(n2), which only works for complex amplitudes.

**Objective Reduction**
A new mechanism for objective reduction [29.30] is also indicated by the way $\pi$ enters the theory as a maximum anti-phase amount comprising part of the maximal perturbation propagation. Consider in the context where there is a 'classical' trigintaduonion path in a congruence of paths (a flow-line description). On the classical path in the congruences, we have $\alpha$ calculated using a $+\pi$ maximal anti-phase, but this could also occur with $-\pi$ maximal anti-phase as well, thus a $\pm\pi$ phase toggle when a zero divisor is encountered in the 32D propagation may be indicated (given the perturbations extending outside the 10D somewhat into the entire 32D). The zero-divisor discontinuity requires the field to reformulate a new 'consistency' with the 32D algebraic propagation (and 64D and higher, as well), with

the result that since the prior $\pi$ phase had the discontinuity, then it must toggle to the other, negative, phase, e.g., objective reduction may occur as a zero-divisor phase-toggle event.

Have zero divisors in the Bi-sed's during interactions on perturbation extensions into 32D. Thus have zero divisor events that may be what has been argued in the case of objective wave collapse (or partial collapse). Thus, an objective reduction mechanism is indicated. The surrounding 32D perturbation 'field' of values is non-zero, so what is suggested is that the $+\pi$ phase toggles to $-\pi$ phase and the field spawns a new propagation consistent with $-\pi$ phase.

**The emergence of 22 parameter theories and representations**
Recall that maximal information propagation occurs for a 10-dimensional doubly-chiral subspace of the 32-dimensional trigintaduonions. The 22 'fixed' dimensions then appear as 22 parameters that 'imprint' on any gauge theory that may emerge from the 10-dimensional propagation (4-dim for space time, 6-dim for a gauge). So, the propagating theory has 22 emergent parameters. Now consider that 10-dim propagation and allow a small perturbation into all dimensions of propagation (including the 22 dimensions). The maximum magnitude of the overall perturbation allowed for unit-norm right (or left) propagation is $\alpha$=(1/137.0359998), where 137 independent tri-octonionic 'braids' comprise the flow of information within the doubly-chiral subspace of the trigintaduonions. These same numbers {10,22,32,137} appear in a number of ancient numerological systems, could it be that they've also identified this maximal information propagation construct in a text analytics setting. Further discussion along these lines is given at the end of [31].

Consider the 22 parameters involved in physics information propagation. From the standard model we see there are 19 parameters required, and their model has known discrepancies, most notably, it doesn't account for the real (or effective) mass of neutrinos, and it doesn't account for dark matter. Suppose we either throw in masses for the neutrinos (they have to be very small to have not been seen yet), or we throw in a new class of massive neutrinos that only interact with the other neutrinos via mix, endowing each with a mass thereby. Alternatively, another Higgs mechanism could be invoked for an effective mass effect to cloak the massless neutrinos that way. Whatever the case, let's suppose the answer to how to complete the standard models falls within one of these cases, where we have the three neutrinos endowed with a mass, one way or another, with a four parameter set to describe neutrino oscillations (mixings), as was done with the quarks. So we now have 19+3+4=26 parameters. Let's now account for the Koide relation found to exist on all the different families (quark and leptonic) of three masses, showing that one mass in a family can be determined once the other two are specified. It isn't understood why the Koide relation should exist, and provide a constraint on the parameters in the extended standard model, but from the perspective of the emergent theory having 22 independent parameters, it makes sense that this would be the case as 26-4=22 results with these relations accounted for. How does $\alpha$ itself fit into this description? Clearly it is an (emergent) maximal limit on the allowed perturbation, so might be expected to appear in a coupling constant somewhere. (This is not necessary, however, as the emergent 22 parameters occur with or without the added perturbation.) It turns out in the standard model, however, there is a relation involving two of the coupling constants to $\alpha$. In other words, two of the parameters are dependent on each other if $\alpha$ is known ($\alpha = (1/4\pi)\, (g_1)^2(g_2)^2/((g_1)^2+(g_2)^2)$, where $g_1$ and $g_2$ are two coupling parameters from the Standard Model). Thus there is one less free parameter and we are at 22-1=21 parameters. The complete model hypothesized for all matter (light and dark) still doesn't include the gravitational coupling constant 'G', so now back to 22, and we appear to be done. The key paramaters *of the propagation*, numbering 22, are now identified. There are still two central parameters from physics not on this list, however, Planck's constant and Boltzmann's constant.

So, is our book-keeping on 22 propagation parameters working if Planck's constant and Boltzmann's constant are not accounted for? Returning to the mathematical object itself and the 'process' of the propagation construct (e.g., repeated right multiplication by a randomly generated unit norm doubly-chiral 'step'). In this picture the steps eventually comprise a path, with the 'path' itself emergent as a 10-dim propagation with chirality. Clearly an internal perception of time in this construct (our perspective) is itself an emergent construct. Even so, there might still be artifacts of a notion of time, or a limit on 'time' granularity, due to the process of the propagation construct. Perhaps, just as law divides into substantive law and procedural law, so too does physical law in the unified propagator theory. Consider that Planck's constant relates to time directly, as the coupling constant relating energy to frequency (a la Einstein), as well as a measure of minimal quanta of angular momentum, and as the non-zero part of the commutation of two complementary quantum variables. Planck's constant goes to the heart of the nature of quanta and the emergent coupling of time to energy. So, Planck's constant pertains to emergent structure due to the right multiplication step with maximal perturbation allowing unit-norm, and how this is seen in the emergent construct. Or more precisely, the unchanging repeated application of the generative ('receiving') right-multiplication 'step' is perceived in the emergent geometry and gauge system as a limit on mimimum quanta for non-zero angular momentum, and other granularity, that is Planck's constant. Similarly, Boltzmann's constant relates (kinetic) energy to temperature. Complex time relates to an inverse temperature in quantum field theory, thus, Boltzmann's constant, like Planck's, may relate to parameters emerging due to the procedural multi-path aspect (and elaborations on emergent constructs in this regard may eventually explain why the theory should be "euclideanizable", for now we will just assume this to be the case as an emergent aspect, although it should be provable on its own accord). Perhaps Plank's constant can be thought of as playing a pivotal role in a "Big Bang" emergent phase where the right multiplication step eventually resolves into collections comprising 'time steps' and then, eventually, time 'flow' as a continuum. Likewise, perhaps Boltzmann's constant can be seen as playing a key role in a thermally equilibrated cosmologically emergent process where Euclideanization is a valid (equilibration) relation.

Counting dimensions involved in maximal propagation gives one set of numbers, while counting the degrees of freedom possible (functionally) given that dimensionality is a different matter. First to recap the dimensionalities arrived at, starting with the Cayley Algebra progression of dimensionalities, and partitions therein, during the process of arriving at maximal information propagation:

$$\infty \rightarrow 2^{\infty} \rightarrow 2^5=32 \text{ (the trigintaduonions)} \rightarrow 32 = 10^* + 22,$$

where we have emergent 22 fixed parameters and chiral propagation in dim = 10* = 4* + 6, where the asterisk denotes the emergent, specific, spacetime reference within the emergent 10D propagation, e.g., we have 4D Lorentzian spacetime with 6D gauge. The 10D propagation is doubly-chiral in the 32 dimensional space, first at the level of the 16 dimensional sedenion subspace, then on the entire 32 dim. trigintaduonion space. The four chiralities of propagation share the same 22 fixed parameters and the same 4D Lorentzian reference frame, thus a 'full' (bosonic) propagation including all four chiralities would have the shared 22 and 4 dimensionalities (or parameters) but with freedom on 6 gauge dimensions for each of the four chiralities. With the discussion on a full propagation involving all chiralites the focus shifts from a counting on dimensions to a counting on functional degrees of freedom (for describing propagation) in that dimensional framework. Any function of a particular parameter (spacetime or gauge) will have independent forward or backward propagations, thus counting for two degrees of freedom each, together with the fixed 22 parameters this give as the number of degrees of freedom at:

Dimensions: single chiral: $10^* + 22 \rightarrow 4^* + 6 + 22$
Dimensions: full doubly-chiral set: $4^* + 4(6) + 22 = 4(7) + 22$
Degrees-of-Freedom: full doubly-chiral set: $2(4)(7) + 22 = (4)(14) + 22 = 56 + 22 = 78$

**Oddities of the non-integer part of $1/\alpha$ – The $11^{th}$ Sefiroth**

In the Results a derivation is given of 137 ‘complete’ braid transmission channels for propagating information, and of a partial 138 braid channel (that is pure imaginary). If we take the experimental result from QED, we know the magnitude of the contribution from the $138^{th}$ channel to be 0.0359907. The appearance of a complex value in association with ‘time’ typically appears in thermodynamics and statistical mechanics, and is an underlying construct in thermal quantum field theory. So the appearance of a small complex contribution is no minor thing as it may provide the explanation for why the variety of euclideanization methods work, and may provide a better understanding of time (especially in quantum systems). A partial channel of transmission of information, perturbative, pure imaginary, thus not only occurs, but may play an integral role in tying time and thermality into the formalism. This might be thought of as a 11 dimension of transmission (akin to the 11 Sefiroth discussed in [31]).

**Conclusion**

Maximal information propagation as an emergent construct appears to require two from of propagation, an early hypercomplex ‘emanation’ that reduces to a chiral 10D propagation in a 32D trigintaduonion space, and standard propagation with complex propagators (consistent with the quantum deFinetti relation [28]) operating inside that 10D propagation of geometry and gauge field. From the ‘emanation’ stage we see the maximum dimensionality and fractal limits provide the fundamental constants that then imprints upon the emergent geometry and gauge field, including giving rise to the constants $\alpha$ and $\pi$.

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**APPENDIX**
**Background on Cayley Algebras**

The list representation for hypercomplex numbers will make things clearer in what follows so will be introduced here for the first seven Cayley algebras:

Reals: $X_0 \rightarrow (X_0)$ .
Complex: $(X_0 + X_1\, \boldsymbol{i}) \rightarrow (X_0, X_1)$ .
Quaternions: $(X_0 + X_1\, \boldsymbol{i} + X_2\, \boldsymbol{j} + X_3\, \boldsymbol{k}) \rightarrow (X_0, X_1, X_2, X_3) \rightarrow (X_0, \ldots, X_3)$ .
Octonions: $(X_0, \ldots, X_7)$ with seven imaginary numbers.
Sedenions: $(X_0, \ldots, X_{15})$ with fifteen imaginary numbers.
Trigintaduonions (a.k.a Bi-Sedenions): $(X_0, \ldots, X_{31})$ with 31 imaginary numbers.
Bi-Trigintaduonions: $(X_0, \ldots, X_{63})$ with 63 types of imaginary number.

Consider how the familiar complex numbers can be generated from two real numbers with the introduction of a single imaginary number '***i***', $\{X_0, X_1\} \rightarrow (X_0 + X_1\, \boldsymbol{i})$. This construction process can be iterated, using two complex numbers, $\{Z_0, Z_1\}$ , and a new imaginary number '***j***':

$(Z_0 + Z_1\, \boldsymbol{j}) = (A+B\boldsymbol{i}) + (C+D\boldsymbol{i})\,\boldsymbol{j} = A+B\boldsymbol{i} + C\boldsymbol{j} + D\boldsymbol{ij} = A+B\boldsymbol{i} + C\boldsymbol{j} + D\boldsymbol{k}$,

where we have introduced a third imaginary number '***k***' where '***ij=k***'. In list notation this appears as the simple rule ((A,B),(C,D)) = (A,B,C,D). This iterative construction process can be repeated, generating algebras doubling in dimensionality at each iteration, to generate the 1,2,4,8,16, 32, and 64 dimensional algebras listed above. The process continues indefinitely to higher orders beyond that, doubling in dimension at each iteration, but we will see that the main algebras of interest for physics are those with dimension 1,2,4,and 8, and sub-spaces of those with dimension 16 and 32 dimensional algebras.

Addition of hypercomplex numbers is done component-wise, so is straightforward. For hypercomplex multiplication, list notation makes the freedom for group splittings more apparent, where any hypercomplex product ZxQ to be expressed as (U,V)x(R,S) by splitting Z=(U,V) and Q=(R,S). This is important because the product rule, generalized by Cayley, uses the splitting capability. The Cayley algebra multiplication rule is:

$$\mathbf{(A,B)(C,D) = ([AC - D^*B],[BC^* + DA])},$$

where conjugation of a hypercomplex number flips the signs of all of its imaginary components:

$$(A,B)^* = \mathrm{Conj}(A,B) = (A^*, -B)$$

The specification of new algebras, with addition and multiplication rules as indicated by the constructive process above, is known as the Cayley-Dickson construction, and this gives rise to what is referred to as the Cayley algebras in what follows.

If you use the Cayley-Dickson procedure to double the octonions to get the sedenions, you retain the properties
common to all Cayley-Dickson algebras [32]:

centrality:	if xy = yx for all y in the algebra A, then x is in the base field of A, which is the real numbers R;
simplicity:	no ideal K other than {0} and the algebra A, or, equivalently, if for all x in K and for all y in A	xy and yx are in K, then K = {0} or A;
flexibility:	(x,y,z) = (xy)z - x(yz) = -(z,y,x), or, equivalently, (xy)x = x(yx) = xyx;
power-associativity:	(xx)x = x(xx) and ((xx)x)x = (xx)(xx), or, equivalently, x^m x^n = x^(m+n);
Jordan-admissibility: xoy = (1/2)(xy + yx) makes a Jordan algebra;
degree two: xx - t(x)x + n(x) = 0, for some real numbers t(x) and n(x);
derivation algebra G2 for octonions and beyond; and
squares of basic units = -1.

For sedenions, you lose the following properties:
(1) the division algebra (over R) property xy = 0 only if x ≠ 0 and y ≠ 0.
(A concrete example of zero divisors in terms of that basis is given by [33]:
(e1 + e10)(e15 - e4) = -e14 - e5 + e5 + e14 = 0.).
(2) linear alternativity: (x,y,z) = (xy)z - x(yz) = (-1)P(Px,Py,Pz), where P is a permutation of sign (-1)P.
and
(3) the Moufang identities: (xy)(zx) = x(yz)x; (xyx)z = x(y(xz)); z(xyx) = ((zx)y)x.

For sedenions, you retain the following properties:
(1) anticommutativity of basic units: xy = -yx;
and
(2) nonlinear alternativity of basic units: (xx)y = x(xy) and (xy)y = x(yy).

**The Formulation of the Problem for Sedenion Propagation**

Further theoretical details on hypercomplex numbers can be found at [34,35]. In what follows multiplications involving unit norm Cayley numbers will be done at the various orders using the Cayley algebra multiplication rule described above, that reduces the order of hypercomplex complex multiplication, which when iterated allows all hypercomplex products to reduce to a collection of Real multiplications. Millions of repeated hypercomplex multiplications are done computationally to demonstrate unit norm propagation in the situations that follow, where B denotes a bisedenion, S denotes a sedenion, O a octonion, Q a quaternion, C for complex, and R for a real:

Sedenions have two unit norm propagators of the form:

S(unit norm) × S(unit norm propagator) = S(unit norm)
S(unit norm) = $S_1(O_{Left},O_{Right}) = S_1(O_L,O_R) = (O_{1L},O_{1R})$

If $S_1$ is unit norm, then norm($S_1$) = $S_1 \times S_1^* = 1$, which for our notation means:
$1 = (O_{1L},O_{1R})\times(O_{1L}^*,-O_{1R}) = ( [O_{1L}\times O_{1L}^*+O_{1R}^*\times O_{1R}], [-O_{1R}\times O_{1L}+O_{1R}\times O_{1L}] )$
$1 = ( [\mathrm{norm}(O_{1L})+\mathrm{norm}(O_{1R})], 0 )$
$1 = \mathrm{norm}(O_{1L})+\mathrm{norm}(O_{1R})$

S(unit norm propagator) = $S_2(O_{Left},O_{Real})$ = $(O_{2L}, \alpha)$ for the right octonion real, e.g., in list notation have $O_{Real}$ = $(\alpha,0,0,0,0,0,0,0)$, so have $(O_{2L}, (\alpha,0,0,0,0,0,0,0))$ which is abbreviated as $(O_{2L}, \alpha)$ where it is understood that $\alpha$ is real and is the real part of the purely real right octonion. There is another type of unit norm propagator where we have $(O_{Real},O_{Right})$ where the same results hold, but the example that follows will use the $(O_{2L}, \alpha)$ form.

If $S_2$ is unit norm, then $\text{norm}(S_2) = S_2 \times S_2^* = 1$, which for our notation means:
$1 = \text{norm}(O_{2L})+\alpha^2$

So we can now ask the question,
Does S(unit norm) × S(unit norm propagator), return a unit norm Sedenion when using the special class of unit norm propagators indicated?

**Proof that Norm($S_1 \times S_2$)=1**

$(S_1\times S_2)=(O_{1L},O_{1R})\times(O_{2L}, \alpha) = ( [O_{1L}\times O_{2L}- \alpha O_{1R}], [\alpha O_{1L}+O_{1R}\times O_{2L}^*] )$
$(S_1\times S_2)^* = ( [O_{1L}\times O_{2L}- \alpha O_{1R}]^*, -[\alpha O_{1L}+O_{1R}\times O_{2L}^*] )$

$\text{norm}(S_1\times S_2) = (S_1\times S_2) \times (S_1\times S_2)^*$
$= ( [O_{1L}\times O_{2L}- \alpha O_{1R}] \times [O_{1L}\times O_{2L}- \alpha O_{1R}]^* +[\alpha O_{1L}+O_{1R}\times O_{2L}^*]^* \times [\alpha O_{1L}+O_{1R}\times O_{2L}^*],$
$-[\alpha O_{1L}+O_{1R}\times O_{2L}^*] \times [O_{1L}\times O_{2L}- \alpha O_{1R}] + [\alpha O_{1L}+O_{1R}\times O_{2L}^*] \times [O_{1L}\times O_{2L}- \alpha O_{1R}])$

$= ( \text{norm}(O_{1L}\times O_{2L})+\text{norm}(O_{1R}\times O_{2L}^*)+\alpha^2 \text{norm}(O_{1R})+\alpha^2 \text{norm}(O_{1L})$
$-\alpha(O_{1L}\times O_{2L})\times O_{1R}^*-\alpha O_{1R}\times(O_{1L}\times O_{2L})^*+\alpha O_{1L}^*\times(O_{1R}\times O_{2L}^*)+\alpha(O_{1R}\times O_{2L}^*)^*\times O_{1L}, \mathbf{0})$

Multiplying the expressions previously obtained, $1 = \text{norm}(O_{1L})+\text{norm}(O_{1R})$ with $1 = \text{norm}(O_{2L})+\alpha^2$, and making use of the norm property norm(xy)=norm(x)norm(y), we have:

$$\text{norm}(S_1\times S_2) = (1-\alpha Z, 0), \text{ where,}$$
$$Z = +(O_{1L}\times O_{2L})\times O_{1R}^*+O_{1R}\times(O_{1L}\times O_{2L})^*-O_{1L}^*\times(O_{1R}\times O_{2L}^*)-(O_{1R}\times O_{2L}^*)^*\times O_{1L}.$$

Since we are computing the norm, which returns only the real component, we know Z must be real. To work with this expression with a little more clarity, switch to the notation:

$A=O_{1L}$; $B=O_{2L}$; $C=O_{1R}^*$, then have
$Z = (A\times B)\times C + C^*\times(A\times B)^*-A^*\times(C^*\times B^*)-(C^*\times B^*)^*\times A$
$Z = (A\times B)\times C + C^*\times(A\times B)^*-A^*\times(B\times C)^*-(B\times C)\times A$

The Cayley algebras up to octionic are also known as the composition algebras for which a number of properties exist. We need the braid laws to proceed, so let's briefly detour to address that. The fundamental composition rule is simply that of the norm of a product being the product of the norms: norm(XY) = norm(X) x normY) Consider the norm of two things added:

$\text{Norm}(X+Y) = (X+Y)(X+Y)^* = XX^* +XY^*+YX^*+YY^*$
$= \text{norm}(X) + \text{norm}(Y) + 2\ \text{real}(XY^*)$

Define [X,Y] = real(XY*) = [norm(X+Y)–norm(X)–norm(Y)]/2, then have another way to express conjugation using norms and real parts:

X* = 2[X,1]-X = 2real(X)-X = (real(X) unchanged, imag(X) negated))

The composition algebras (up to octionic) build from the core norm(XY) = norm(X) x normY) relation to arrive at a number of interesting properties, including the 'braid' laws: [XY,Z] = [Y,X*Z] and [XY,Z]=[X,ZY*]. To arrive at the Braid law (following [34]) you start with the composition law norm(XY)=norm(X)norm(Y), you then prove the scaling law, [XY,XZ]=norm(x)[Y,Z], by substituting Y with Y+Z in the composition law. Then establish the exchange law [XY,UZ] = 2[X,U][Y,Z]-[XZ,UY] by substituting X with X+U in the scaling law. If you put U=1 in the exchange law, it reduces to forms allowing the braid law to be shown.
Let's apply the braid law for the form [XY,Z] to the (B×C)×A term, so let's look at the braid law for [BC,A*] = [C,B*A*], which can be rewritten as:

norm(BC+A*)–norm(BC)–norm(A*) = norm(C+B*A*)–norm(C)–norm(B*A*)
norm(BC+A*)=norm(BC)+norm(A*)+(BC)A+A*(BC)*
norm(C+B*A*)=norm(C)+norm(B*A*)+C(AB)+(AB)*C*

putting this together: (BC)A+A*(BC)*=C(AB)+(AB)*C*. So we can now rewrite the (B×C)×A term as: (B×C)×A = C×(A×B)+(A×B)*×C*–A*×(B×C)*. Substituting this back into Z:

Z = (A×B)×C + C*×(A×B)*–C×(A×B)–(A×B)*×C*
= [(A×B)×C–C×(A×B)] + [C*×(A×B)*–(A×B)*×C*]

What is a commutator on the Cayley numbers, is it necessarily non-real?

XY=(A,B)(C,D) = ([AC–D*B],[BC*+DA])
YX=(C,D)(A,B)= ([CA–B*D],[DA*+BC])
{X,Y}=XY-YX= ([AC–CA+B*D–D*B],[BC*–BC+DA–DA*])
{X,Y}= ( [{A,C}+2Im(B*D)], [B 2Im(C) + D 2Im(A)] )

So the commutator at one order of Cayley number is reduced to an expression involving the commutator at the next lower order Cayley number, plus a bunch of other terms that don't contribute to the real component. This can be iterated to arrive at the real algebra in the commutator, where the commutator is zero, thereby establishing that the commutator on the Cayley numbers must result in a pure imaginary Cayley number. This being the case, we see that since Z consists of two commutator terms, neither of which has a real contribution, and since Z must be real, this proves that Z=0.

This proves the first extension, for unit-norm propagators that are Sedenions of the form $S_{Left}=(O_{Left},\alpha)$ or $S_{Right}=(\alpha,O_{Right})$, where $O_{Left}$ and $O_{Right}$ are any octonion. The next extension is to unit-norm propagators that are Bisedenion by using similar constructions, e.g., Bisedenions, of the form $B=(S_{Left},S_{Real}) = ( (O_{Left},\alpha) , \beta )$. (Note that $\alpha$ is a real octonion, while $\beta$ is a purely real sedenion.)

**The Formulation of the Problem for Bi-Sedenion Propagation**

Bisedenions have two unit norm propagators of the form:
B(unit norm) × B(unit norm propagator) = B(unit norm)
B(unit norm) = $B_1(S_{Left},S_{Right}) = B_1(S_L,S_R) = (S_{1L},S_{1R})$

If $B_1$ is unit norm, then norm($B_1$) = $B_1 \times B_1^* = 1$, which for our notation means:
$1 = (S_{1L},S_{1R})\times(S_{1L}^*,-S_{1R}) = ( [S_{1L}\times S_{1L}^*+S_{1R}^*\times S_{1R}], [-S_{1R}\times S_{1L}+S_{1R}\times S_{1L}] )$
$1 = ( [\text{norm}(S_{1L})+\text{norm}(S_{1R})], 0 )$
$1 = \text{norm}(S_{1L})+\text{norm}(S_{1R})$

B(unit norm propagator) = $B_2(S_{Left},S_{Real}) = (S_{2L}, \beta)$ for the right sedenion real, e.g., in list notation have $S_{Real} = (\beta,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)$, so have $(O_{2L}, (\beta,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0))$ which is abbreviated as $(O_{2L}, \beta)$ where it is understood that $\beta$ is real and is the real part of the purely real right sedenion. There is another type of unit norm propagator where we have $(S_{Real},S_{Right})$ where the same results hold, but the example that follows will use the $(S_{2L}, \beta)$ form.

If $B_2$ is unit norm, then norm($B_2$) = $B_2 \times B_2^* = 1$, which for our notation means:
$1 = \text{norm}(S_{2L})+\beta^2$

So we can now ask the question:
Does B(unit norm) × B(unit norm propagator), return a unit norm Bisedenion when using the special class of unit norm propagators indicated?

**Proof that Norm($B_1\times B_2$)=1**

$(B_1\times B_2)=(S_{1L},S_{1R})\times(S_{2L}, \beta) = ( [S_{1L}\times S_{2L}- \beta S_{1R}], [\beta S_{1L}+S_{1R}\times S_{2L}^*] )$
$(B_1\times B_2)^* = ( [S_{1L}\times S_{2L}- \beta S_{1R}]^*, -[\beta S_{1L}+S_{1R}\times S_{2L}^*] )$

$\text{norm}(B_1\times B_2) = (B_1\times B_2) \times (B_1\times B_2)^*$
$= ( [S_{1L}\times S_{2L}- \beta S_{1R}] \times [S_{1L}\times S_{2L}- \beta S_{1R}]^* +[\beta S_{1L}+S_{1R}\times S_{2L}^*]^* \times [\beta S_{1L}+S_{1R}\times S_{2L}^*],$
$-[\beta S_{1L}+S_{1R}\times S_{2L}^*] \times [S_{1L}\times S_{2L}- \beta S_{1R}] + [\beta S_{1L}+S_{1R}\times S_{2L}^*] \times [S_{1L}\times S_{2L}- \beta S_{1R}])$

$= ( \text{norm}(S_{1L}\times S_{2L})+\text{norm}(S_{1R}\times S_{2L}^*)+\beta^2\, \text{norm}(S_{1R})+\beta^2\, \text{norm}(S_{1L})$
$-\beta(S_{1L}\times S_{2L})\times S_{1R}^*-\beta S_{1R}\times(S_{1L}\times S_{2L})^*+\beta S_{1L}^*\times(S_{1R}\times S_{2L}^*)+\beta(S_{1R}\times S_{2L}^*)^*\times S_{1L}, \mathbf{0})$

To proceed as before we need to show that the norm property norm(xy) = norm(x)norm(y) holds for the sedenions when one of them is constrained to be in the form of the sedenion propagator, e.g., does $\text{norm}(S_{1L}\times S_{2L})=\text{norm}(S_{1L})\times\text{norm}(S_{2L})$ where $S_{2L}$ is in the form of the sedenion propagator?

$\text{norm}(S_{1L}\times S_{2L}) = (S_{1L}\times S_{2L}) \times (S_{1L}\times S_{2L})^*$
$= ( [O_{1LL}\times O_{2LL}- \alpha O_{1LR}] \times [O_{1LL}\times O_{2LL}- \alpha O_{1LR}]^* +$
$[\alpha O_{1LL}+O_{1LR}\times O_{2LL}^*]^* \times [\alpha O_{1LL}+O_{1LR}\times O_{2LL}^*],$
$-[\alpha O_{1LL}+O_{1LR}\times O_{2LL}^*] \times [O_{1LL}\times O_{2LL}- \alpha O_{1LR}] +$
$[\alpha O_{1LL}+O_{1LR}\times O_{2LL}^*] \times [O_{1LL}\times O_{2LL}- \alpha O_{1LR}])$

$= (\ \text{norm}(O_{1LL}\times O_{2LL})+\text{norm}(O_{1LR}\times O_{2LL}^*)+\alpha^2\ \text{norm}(O_{1LR})+\alpha^2\ \text{norm}(O_{1LL})-$
$\quad \alpha(O_{1LL}\times O_{2LL})\times O_{1LR}^*-\alpha O_{1LR}\times(O_{1LL}\times O_{2LL})^*+$
$\quad \alpha O_{1LL}^*\times(O_{1LR}\times O_{2LL}^*)+\alpha(O_{1LR}\times O_{2LL}^*)^*\times O_{1LL},\quad \mathbf{0})$

Now that we've reduced to this level, we know that the octonions will offer the standard norm property whereby $\text{norm}(O_{1LL}\times O_{2LL})=\text{norm}(O_{1LL})\text{norm}(O_{2LL})$ and we show the other terms are zero since real yet consisting of commutators, the latter arrangements made possible by manipulations according to the braid laws that hold for the composition algebras (including the octonions) without restriction.

So as before, by multiplying the expressions previously obtained, $1 = \text{norm}(S_{1L}) + \text{norm}(S_{1R})$ with $1 = \text{norm}(S_{2L})+\beta^2$, and making use of the norm property norm(xy)=norm(x)norm(y) applicable for the terms of interest, we have:

$$\text{norm}(B_1\times B_2) = (1-\beta Z,\ 0),\ \text{where,}$$
$$Z = +(S_{1L}\times S_{2L})\times S_{1R}^*+S_{1R}\times(S_{1L}\times S_{2L})^*-S_{1L}^*\times(S_{1R}\times S_{2L}^*)-(S_{1R}\times S_{2L}^*)^*\times S_{1L}.$$

Since we are computing the norm, which returns only the real component, we know Z must be real. As with the lower order Cayley extension, we need the braid laws to proceed at this juncture.

What is $(S_{1L}\times S_{2L})\times S_{1R}^*$ when accounting for the special form of $S_{2L}=(O_{2LL},\ \alpha)$? First calculate $(S_{1L}\times S_{2L})$:

$(S_{1L}\times S_{2L})=(O_{1LL},O_{1LR})(O_{2LL},\ \alpha) = (\ [O_{1LL}\times O_{2LL}-\ \alpha O_{1LR}],\ [\alpha O_{1LL}+O_{1LR}\times O_{2LL}^*]\ )$

Then
$(S_{1L}\times S_{2L})\times S_{1R}^* = (\ [O_{1LL}\times O_{2LL}-\ \alpha O_{1LR}],\ [\alpha O_{1LL}+O_{1LR}\times O_{2LL}^*]\ )\ (O_{1RL}^*,-O_{1RR})$
$= ([O_{1LL}\times O_{2LL}-\ \alpha O_{1LR}]O_{1RL}^* + O_{1RR}^*[\alpha O_{1LL}+O_{1LR}\times O_{2LL}^*],$
$\quad -O_{1RR}\ [O_{1LL}\times O_{2LL}-\ \alpha O_{1LR}] + [\alpha O_{1LL}+O_{1LR}\times O_{2LL}^*]O_{1RL})$
$= (\ (O_{1LL}\times O_{2LL})\times O_{1RL}^*-\ \alpha O_{1LR}\times O_{1RL}^*+\alpha\ O_{1RR}^*\times O_{1LL}+\ O_{1RR}^*\times(O_{1LR}\times O_{2LL}^*),$
$\quad -O_{1RR}\ \times(O_{1LL}\times O_{2LL})+\alpha O_{1RR}\times O_{1LR}+\alpha O_{1LL}\times O_{1RL}+(O_{1LR}\times O_{2LL}^*)\times O_{1RL}\ )$

$S_{1R}\times(S_{1L}\times S_{2L})^* = (O_{1RL},O_{1RR})\ \ (\ [O_{1LL}\times O_{2LL}-\ \alpha O_{1LR}]^*,\ -[\alpha O_{1LL}+O_{1LR}\times O_{2LL}^*]\ )$
$= (\ O_{1RL}\times[(O_{1LL}\times O_{2LL})^*-\ \alpha O_{1LR}^*]+[\alpha O_{1LL}^*+(O_{1LR}\times O_{2LL}^*)^*]\times O_{1RR},$
$\quad -[\alpha O_{1LL}+O_{1LR}\times O_{2LL}^*]\times O_{1RL}+O_{1RR}\times[O_{1LL}\times O_{2LL}-\ \alpha O_{1LR}]\ )$
$= (\ O_{1RL}\times(O_{1LL}\times O_{2LL})^*-\ \alpha O_{1RL}\times O_{1LR}^*\ +\alpha O_{1LL}^*\times O_{1RR}+(O_{1LR}\times O_{2LL}^*)^*\times O_{1RR}\ ,$
$\quad -\alpha O_{1LL}\times O_{1RL}-(O_{1LR}\times O_{2LL}^*)\times O_{1RL}+O_{1RR}\times(O_{1LL}\times O_{2LL})-\ \alpha O_{1RR}\times O_{1LR}\ )$

Putting these first two terms together:
$+(S_{1L}\times S_{2L})\times S_{1R}^*+S_{1R}\times(S_{1L}\times S_{2L})^* =$
$(\ (O_{1LL}\times O_{2LL})\times O_{1RL}^*+O_{1RL}\times(O_{1LL}\times O_{2LL})^*$
$-\ \alpha O_{1LR}\times O_{1RL}^*+\alpha\ O_{1RR}^*\times O_{1LL}-\ \alpha O_{1RL}\times O_{1LR}^*\ +\alpha O_{1LL}^*\times O_{1RR}$
$+\ O_{1RR}^*\times(O_{1LR}\times O_{2LL}^*)+(O_{1LR}\times O_{2LL}^*)^*\times O_{1RR},\qquad 0)$

For $S_{1L}^*\times(S_{1R}\times S_{2L}^*)$ we have:
$(S_{1R}\times S_{2L}^*)=(O_{1RL},O_{1RR})(O_{2LL}^*,\ -\alpha) = ([O_{1RL}\times O_{2LL}^*+\ \alpha O_{1RR}],\ [-\alpha O_{1RL}+O_{1RR}\times O_{2LL}]\ )$

So, $S_{1L}^*\times(S_{1R}\times S_{2L}^*) = (O_{1LL}^*,-O_{1LR})\times([O_{1RL}\times O_{2LL}^*+\alpha O_{1RR}], [-\alpha O_{1RL}+O_{1RR}\times O_{2LL}])$
$= ( O_{1LL}^*\times(O_{1RL}\times O_{2LL}^*)+\alpha O_{1LL}^*\times O_{1RR}-\alpha O_{1RL}^*\times O_{1LR}+(O_{1RR}\times O_{2LL})^*\times O_{1LR}$ , term)
While for $(S_{1R}\times S_{2L}^*)^*\times S_{1L}$ have
$(S_{1R}\times S_{2L}^*)^*\times S_{1L}= ([O_{1RL}\times O_{2LL}^*+ \alpha O_{1RR}]^*, [\alpha O_{1RL}-O_{1RR}\times O_{2LL}])\times(O_{1LL},O_{1LR})$
$= ([O_{1RL}\times O_{2LL}^*+ \alpha O_{1RR}]^*\times O_{1LL}-O_{1LR}^*\times[\alpha O_{1RL}-O_{1RR}\times O_{2LL}]$, - term)

$S_{1L}^*\times(S_{1R}\times S_{2L}^*)+(S_{1R}\times S_{2L}^*)^*\times S_{1L}=$
$( O_{1LL}^*\times(O_{1RL}\times O_{2LL}^*)+(O_{1RL}\times O_{2LL}^*)^*\times O_{1LL}$
$+\alpha O_{1LL}^*\times O_{1RR}-\alpha O_{1RL}^*\times O_{1LR}+ \alpha O_{1RR}^*\times O_{1LL}-\alpha O_{1LR}^*\times O_{1RL}$
$+(O_{1RR}\times O_{2LL})^*\times O_{1LR}+O_{1LR}^*\times(O_{1RR}\times O_{2LL}),\quad 0)$

So have,
$Z = ( \quad \{(O_{1LL}\times O_{2LL})\times O_{1RL}^*+O_{1RL}\times(O_{1LL}\times O_{2LL})^*$
$\quad -O_{1LL}^*\times(O_{1RL}\times O_{2LL}^*)-(O_{1RL}\times O_{2LL}^*)^*\times O_{1LL}\}$ +
$\quad \{O_{1RR}^*\times(O_{1LR}\times O_{2LL}^*)+(O_{1LR}\times O_{2LL}^*)^*\times O_{1RR}$
$\quad -(O_{1RR}\times O_{2LL})^*\times O_{1LR}-O_{1LR}^*\times(O_{1RR}\times O_{2LL})\}$ +
$- \alpha O_{1LR}\times O_{1RL}^*+\alpha O_{1RR}^*\times O_{1LL}- \alpha O_{1RL}\times O_{1LR}^* +\alpha O_{1LL}^*\times O_{1RR}$
$-\alpha O_{1LL}^*\times O_{1RR}+\alpha O_{1RL}^*\times O_{1LR}-\alpha O_{1RR}^*\times O_{1LL}+\alpha O_{1LR}^*\times O_{1RL}$ , 0)

$Z = (\{im\}+\alpha\{O_{1RL}^*\times O_{1LR}+\alpha O_{1LR}^*\times O_{1RL}- \alpha O_{1LR}\times O_{1RL}^*- \alpha O_{1RL}\times O_{1LR}^*\}$ , 0)
$Z = (\{im\}+\alpha\{2Im\{O_{1RL}^*\times O_{1LR}\} +2Im\{O_{1LR}^*\times O_{1RL}\}$ , 0)

So again, have that Z = pure imaginary, and since it must be real, it is thus zero.
Thus, we have $norm(B_1\times B_2) = 1$. This proves the second extension, for unit-norm propagators that are Bisedenions of the form $B_{Left}=(S_{Left},\beta)$ or $B_{Right}=(\beta,S_{Right})$, where $S_{Left}$ and $S_{Right}$ are sedenion propagators shown in the first extension, e.g., $S_{Left}=(O_{Left},\alpha)$. (Note that $\alpha$ is a purely real octonion, while $\beta$ is a purely real sedenion.)

**The Formulation of the Problem for Bi-trigintaduonion Propagation**

After the bisedenions (also known as trigintaduonions) come the bitrigintaduonions, the 64-component Cayley algebra (denoted by 'T' in following but later when I reference the RCHO(ST) hypothesis, the 'T' refers to trigintaduonions). Let's try extending further to see if we can have $norm(T_1\times T_2) = 1$, when we build with a similar extension method to define our unit-norm propagator: $T_{Left}=(B_{Left},\gamma)$ , $B_{Left}=(S_{Left},\beta)$ , and $S_{Left}=(O_{Left},\alpha)$, where, as before, once we get to the octionic Cayley level we are unrestricted (e.g., $O_{Left}$ can be any octonion). Let's see if we can construct, as before, a T unit norm propagators of the form:

T(unit norm) × T(unit norm propagator) = T(unit norm)
T(unit norm) = $T_1(B_{Left},B_{Right}) = T_1(B_L,B_R) = (B_{1L},B_{1R})$

If $T_1$ is unit norm, then $norm(T_1) = T_1 \times T_1^* = 1$, which for our notation means:
$1 = (B_{1L},B_{1R})\times(B_{1L}^*,-B_{1R}) = ( [B_{1L}\times B_{1L}^*+B_{1R}^*\times B_{1R}], [-B_{1R}\times B_{1L}+B_{1R}\times B_{1L}] )$
$1 = ( [norm(B_{1L})+norm(B_{1R})], 0 )$
$1 = norm(B_{1L})+norm(B_{1R})$

T(unit norm propagator) = $T_2(B_{Left},B_{Real}) = (B_{2L}, \gamma)$ for the right bisedenion real $\gamma$ is real and is the real part of the purely real right bisedenion.

If $T_2$ is unit norm, then $\text{norm}(T_2) = T_2 \times T_2^* = 1$, which for our notation means:
$1 = \text{norm}(B_{2L})+\gamma^2$

So we can now ask the question,
Does T(unit norm) × T(unit norm propagator), return a unit norm bitrigintaduonion when using the special class of unit norm propagators indicated?

**Failure of Proof construction for $\text{Norm}(T_1\times T_2)=1$ , and computational proof of failure of $\text{Norm}(T_1\times T_2)=1$**

$(T_1\times T_2)=(B_{1L},B_{1R})\times(B_{2L}, \gamma) = ( [B_{1L}\times B_{2L}- \gamma B_{1R}], [\gamma B_{1L}+B_{1R}\times B_{2L}^*] )$
$(T_1\times T_2)^* = ( [B_{1L}\times B_{2L}- \gamma B_{1R}]^*, -[\gamma B_{1L}+B_{1R}\times B_{2L}^*] )$

$\text{norm}(T_1\times T_2) = (T_1\times T_2) \times (T_1\times T_2)^*$
$= ( [B_{1L}\times B_{2L}- \gamma B_{1R}] \times [B_{1L}\times B_{2L}- \gamma B_{1R}]^* +[\gamma B_{1L}+B_{1R}\times B_{2L}^*]^* \times [\gamma B_{1L}+B_{1R}\times B_{2L}^*],$
$-[\gamma B_{1L}+B_{1R}\times B_{2L}^*] \times [B_{1L}\times B_{2L}- \gamma B_{1R}] + [\gamma B_{1L}+B_{1R}\times B_{2L}^*] \times [B_{1L}\times B_{2L}- \gamma B_{1R}])$

$= ( \text{norm}(B_{1L}\times B_{2L})+\text{norm}(B_{1R}\times B_{2L}^*)+\gamma^2 \text{norm}(B_{1R})+\gamma^2 \text{norm}(B_{1L})$
$-\gamma(B_{1L}\times B_{2L})\times B_{1R}^*-\gamma B_{1R}\times(B_{1L}\times B_{2L})^*+\gamma B_{1L}^*\times(B_{1R}\times B_{2L}^*)+\gamma(B_{1R}\times B_{2L}^*)^*\times B_{1L}, \mathbf{0})$

To proceed as before we need to show that the norm property norm(xy) = norm(x)norm(y) holds for the bisedenions when one of them is constrained to be in the form of the bisedenion propagator, e.g., does $\text{norm}(B_{1L}\times B_{2L}) = \text{norm}(B_{1L})\times\text{norm}(B_{2L})$ where $B_{2L}$ is in the form of the bisedenion propagator?

$\text{norm}(B_{1L}\times B_{2L}) = (B_{1L}\times B_{2L}) \times (B_{1L}\times B_{2L})^*$
$= ( [S_{1LL}\times S_{2LL}- \beta S_{1LR}] \times [S_{1LL}\times S_{2LL}- \beta S_{1LR}]^* +$
$[\beta S_{1LL}+S_{1LR}\times S_{2LL}^*]^* \times [\beta S_{1LL}+S_{1LR}\times S_{2LL}^*],$
$-[\beta S_{1LL}+S_{1LR}\times S_{2LL}^*] \times [S_{1LL}\times S_{2LL}- \beta S_{1LR}] +$
$[\beta S_{1LL}+S_{1LR}\times S_{2LL}^*] \times [S_{1LL}\times S_{2LL}- \beta S_{1LR}])$

$= ( \text{norm}(S_{1LL}\times S_{2LL})+\text{norm}(S_{1LR}\times S_{2LL}^*)+\beta^2 \text{norm}(S_{1LR})+\beta^2 \text{norm}(S_{1LL})-$
$\beta(S_{1LL}\times S_{2LL})\times S_{1LR}^*-\beta S_{1LR}\times(S_{1LL}\times S_{2LL})^*+$
$\beta S_{1LL}^*\times(S_{1LR}\times S_{2LL}^*)+\beta(S_{1LR}\times S_{2LL}^*)^*\times S_{1LL}, \mathbf{0})$

Now that we've reduced to this level we see there is a problem. In the prior reduction we arrived at the variables being octonions at this stage, for which the norm property and braid laws of the octionoic composition algebra allowed $\text{norm}(O_{1LL}\times O_{2LL}) = \text{norm}(O_{1LL})\text{norm}(O_{2LL})$ and showed the non-norm terms were zero by manipulations using the braid laws that hold for the composition algebras. Now that we've moved to the next higher Cayley algebra's in the derivation, and in our extension construction, we now are asking the sedenions to act as a composition algebra to proceed (on an unrestricted part of the Sedenion algebra). The construction fails. Thus, the extension process does not extend past the Bisedenions, it basically requires the Cayley algebra at two Cayley levels lower to still be a composition algebra. It is still possible to extend to the bisedenions because at two levels lower you still have the octonions, which are a composition algebra as needed. Computationally we see a failure to propagate the bitrigintaduonions so this is consistent.

**Code and Computational Validation**

The key software solution to discover/verify the results computationally is a the recursive Cayley definition for multiplication, which avoids use of lookup tables and avoids commutation and associativity issues encountered at higher order. It is shown next. The cayley subroutine takes the references to any pair of Cayley numbers (represented in list form, so represented as simple arrays), and multiplies those Cayley numbers and returns the Cayley number answer (in list form, thus an array). The main usage was with randomly generated unit norm Cayley numbers that were multiplied (from right) against a "running product". Tests on unit norm hold for millions of running product evaluations in cases where there the unit norm propagations are validated, so, like the perfectly meshed gears of a machine, or the perfectly 'braided' threads of a very long string.

The bignum module was used with 50 decimal places of precision in most experiments, with some experiments at 100 decimal places of precision in further validation testing. Using bignum allows much higher precision handling (needed for the iterative processes of repeated multiplicative updates). The use of bignum, however, entails number representation/storage via strings and is vastly slower than normal arithmetic operations. Furthermore, modern GPU enhancements are not possible with the string handling intermediaries, so the resultant computational threads are CPU intensive and slow.

---------------------------------- cayley_multiplication.pl -------------------------------------------

```
sub cayley {
    my ($ref1,$ref2)=@_;
    my @input1=@{$ref1};
    my @input2=@{$ref2};
    my $order1=scalar(@input1);
    my $order2=scalar(@input2);
    my @output;
    if ($order1 != $order2) {die;}
    if ($order1 == 1) {
        $output[0]=$input1[0]*$input2[0];
    }
    else{
        my @A=@input1[0..$order1/2-1];
        my @B=@input1[$order1/2..$order1-1];
        my @C=@input2[0..$order1/2-1];
        my @D=@input2[$order1/2..$order1-1];
        my @conjD=conj(\@D);
        my @conjC=conj(\@C);
        my @cay1 = cayley(\@A,\@C);
        my @cay2 = cayley(\@conjD,\@B);
        my @cay3 = cayley(\@D,\@A);
        my @cay4 = cayley(\@B,\@conjC);
        my @left;
        my @right;
        my $length = scalar(@cay1);
        my $index;
        for $index (0..$length-1) {
            $left[$index] = $cay1[$index] - $cay2[$index];
            $right[$index] = $cay3[$index] + $cay4[$index];
        }
        @output=(@left,@right);
    }
    return @output;
}
```

---------------------------------- cayley_multiplication.pl -------------------------------------------

**Unit-norm multiplicative 'step' generation method (same as used in [3])**

We now consider a randomly generated propagation step that consists of a unit norm that has a randomly generated small perturbation. Consider the eight element octonion denoted: **$\{\Delta x_0, \delta x_1, \delta x_2, \ldots \delta x_7\}$,** where the real component is **$\Delta x_0 \approx 1$:**

$$(\Delta x_0)^2 = 1 - \Sigma (\delta x_i)^2 ,$$

And where each $\delta x_i$ is generated by a randomly generated number uniformly distributed on the interval (-0.5 .. 0.5), with an additional perturbation-factor '**δ**', e.g., the max magnitude imaginary perturbation from pure real (**$\Delta x_0=1$**), measured with $L_1$ norm, is simply 7 times $\delta/2$ (for seven imaginary components).

For the octonions unrestricted unit norm propagation is possible, i.e., all of the components can be independently generated and then normalized to have $L_2$ norm =1. So, the restriction to **$\Delta x_0 \approx 1$** isn't needed. For the left chiral extension spaces we have:

> Propagating chiral left sedenion: $\{\Delta x_0, \delta x_1, \delta x_2, \ldots \delta x_7, \delta x_8\}$, with $\delta x_9=0, \ldots, \delta x_{15}=0$,
>
> and the
>
> Propagating chiral left bi-sedenion: $\{\Delta x_0, \delta x_1, \delta x_2, \ldots \delta x_7, \delta x_8, \delta x_{16}\}$, with $\delta x_9=0, \ldots, \delta x_{15}=0, \delta x_{17}=0, \ldots, \delta x_{31}=0$,

where, the propagating chiral left bi-sedenion has a small non-propagating component (here **$\delta x_9$** nonzero is chosen), and now we formally require **$\Delta x_0 \approx 1$** on propagation steps:

> Propagating small perturbation chiral left bi-sedenion: $\{\Delta x_0, \delta x_1, \delta x_2, \ldots \delta x_7, \delta x_8, \boldsymbol{\delta x_9}, \delta x_{16}\}$, with $\delta x_{10}=0, \ldots, \delta x_{15}=0, \delta x_{17}=0, \ldots, \delta x_{31}=0$.

For the small perturbation steps that are randomly generated, there are now ten imaginary components, so in what follows, the maximum magnitude of the imaginary components, measured with $L_1$ norm, denoted **Δ,** is **Δ=**$10\delta/2=5\delta$.

**Emergent Parameters**

In the tables that follow are shown the emergent parameters when alpha-perturbations (perturbations with max perturbation α) are injected for each of the 22 non-propagating parameters. Regardless of injection parameter, if perturbation exceeds α, the norm=1 relation fails, and propagation eventually dies with norm ≈ 0. This is to be expected given the identification of α as the max-perturbation limit in [2]. What is odd is that if perturbation is less than α, but still in the vicinity of α, norm=1 behavior appears to eventually fail (after millions of iterations, and using bignum precision) as can be seen in the real component eventually falling to zero. In other words, the iterative procedure underlying the propagator definition, not surprisingly, is giving rise to fractal behavior (and abrupt transitions). I say not surprisingly because the single parameter noise injection that we are using (in repeated multiplicative iterations) is such that we've set up an iterative process with a 1-dim parameter space and are seeing possible fractal behavior -- a well-known phenomenon in 1-D iterative mappings.

The results that follow are *preliminary estimates* [3] on the 'letters of reality' (actually numbers in this numerogenesis algebraic theory) in that both the noise injection method can lead to artifacts, and due to the slow process of doing bi-sedenion multiplication with bignum(50). In the tabulations we consider unit norm chiral bi-sedenion propagation. In particular, we consider unit element chiral bisedenion propagations, with α perturbations introduced, separately, at each of the non-propagating bi-sedenion parameters. If working with a perturbation greater than α we expect the real component to start at one (we begin with a unit element chiral bisedenion) and eventually decay to zero, and cross-over to negative values, as it begins to randomly walk. To a lesser extent, and with fractal structure, this also appears to be true for perturbations introduced that are in the vicinity of α but less than α. For perturbations precisely at 'α', we expect the real component to decay/search for a while, but to then asymptote/lock-on to a particular non-zero (emergent) value with well-defined variance about that asymptote, and never crossing zero. In other words, if we want to propagate one bit of information via the non zero-crossing real component asymptote of the bi-sedenion indefinitely, it is hypothesized that we can do so using α perturbation propagators. In Fig. 4 is shown the Histogram on real components (rc) observations after each multiplicative iteration, where an emergent rc=0.971 appears in the first 60,000 propagation iterations.

To recap, for 'off-shell' bi-sedenion propagation at maximum perturbation amplitude α, we examine the behavior of the real component (rc) of the bi-sedenion. This is because we are effectively describing propagation starting with a unit bi-sedenion (so only have rc=1 nonzero), followed by multiplicative propagation steps by way of bisedenions perturbed by at most the fraction α into the bisedenions 31 imaginary components. We consider each of the 22 possible non-propagating parameters in separate perturbation-at-alpha analyses, where the emergent behavior on the rc value is obtained. For the 'α propagation', where noise injection is solely in a particular non-propagating component, we expect the rc component to decay but to eventually asymptote to a positive value (and never cross zero). The Results in Table 1 show the emergent 22 parameters when propagation is done with precisely α perturbation, where α is taken to be the highest precision value known provided by QED (1/137.035999070) [36] (which appears via δ=0.001459470514006 in the code).

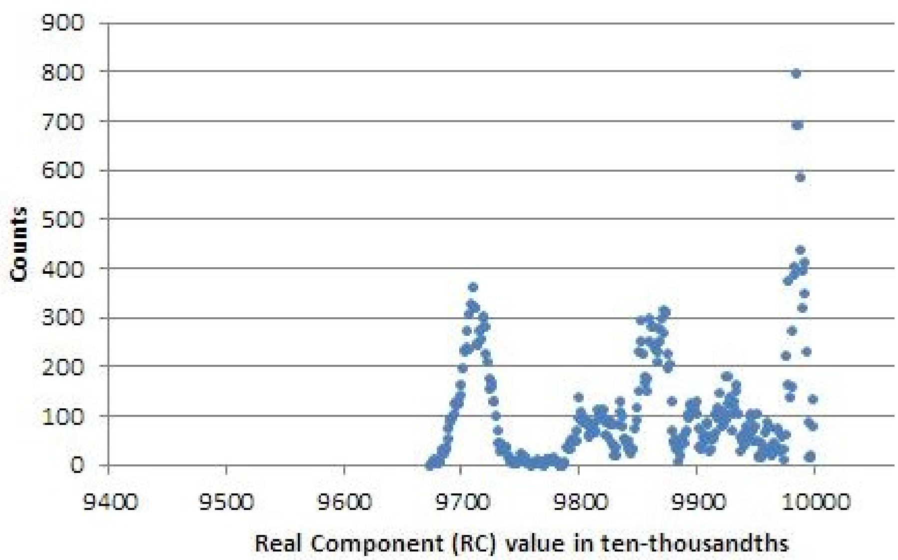


Fig. 4. Histogram of real-component values observed in the first 60,000 iterations of α propagation, where the perturbation parameter is $\delta x_{30}$. As the propagation begins the RC value is at 1.0=10,000/10,000, so at the rightmost dot in the histogram. As the multiplicative operations proceed, the rc value decays thru the range to rc =0.9750, then begins to catch the asymptote with mean at 0.9710.

The $\alpha$ propagations are examined for each of the 22 different non-propagating components, with each taken individually as sole source of non-propagating perturbation in its respective $\alpha$ propagation experiment. Since this process is selecting propagators somewhat arbitrarily, perhaps not as much utility can be extracted from the asymptotic RC values as from their *variance* information. In other words, by injecting noise perturbations into each of the 22 parameters separately (like playing a recorder with only one hole depressed at a time), computational experiments are attempting to arrive at information respective to 22 parameters, but may do so in a mixed form not so useful when expressed in the RC values. Furthermore, the Gaussian distributions that appear to be emergent at the asymptotes have variance values (or their inverses as shown in Table 1) that may provide the most utility. In essence the variance can be thought of as describing a statistical restoring force that's occurring in the bi-sedenion propagation due to the odd properties of bisedenion in general, e.g., they are: non-associative, non-commutative, have zero-divisors, and lack of inverse due to lack of norm. Bisedenion properties are not theoretically fully understood at this time, thus the computational efforts described here (and in [1-3]) to try to resolve matters further.

| Off-shell parameter | Asymptotic Real Component (RC) | Asymptotic RC FWHM | Asympt. RC 1/Variance |
|---|---|---|---|
| $\delta x_9$ | 0.9823 | 0.0047 | ***246,819*** |
| $\delta x_{10}$ | 0.9361 | 0.0044 | ***281,623*** |
| $\delta x_{11}$ | 0.9585 | 0.0030 | ***605,803*** |
| $\delta x_{12}$ | 0.9856 | 0.0021 | ***1,236,332*** |
| $\delta x_{13}$ | 0.9953 | 0.0017 | ***1,886,583*** |
| $\delta x_{14}$ | 0.9343 | 0.0029 | ***648,302*** |
| $\delta x_{15}$ | 0.9745 | 0.0023 | ***1,030,666*** |
| $\delta x_{17}$ | 0.9644 | 0.0039 | ***358,463*** |
| $\delta x_{18}$ | 0.9745 | 0.0050 | ***218,089*** |
| $\delta x_{19}$ | 0.9799 | 0.0060 | ***151,450*** |
| $\delta x_{20}$ | 0.9792 | 0.0053 | ***194,098*** |
| $\delta x_{21}$ | 0.9639 | 0.0048 | ***236,641*** |
| $\delta x_{22}$ | 0.9797 | 0.0028 | ***695,436*** |
| $\delta x_{23}$ | 0.9593 | 0.0037 | ***398,263*** |
| $\delta x_{24}$ | 0.9826 | 0.0066 | ***125,165*** |
| $\delta x_{25}$ | 0.9979 | 0.0012 | ***3,786,267*** |
| $\delta x_{26}$ | 0.9615 | 0.0059 | ***156,628*** |
| $\delta x_{27}$ | 0.9892 | 0.0041 | ***324,344*** |
| $\delta x_{28}$ | 0.9497 | 0.0051 | ***209,620*** |
| $\delta x_{29}$ | 0.9326 | 0.0052 | ***201,635*** |
| $\delta x_{30}$ | 0.9710 | 0.0022 | ***1,126,493*** |
| $\delta x_{31}$ | 0.9706 | 0.0020 | ***1,363,056*** |

**Table 1. The 22 letters of reality.** The 'letters' are emergent real parameters (i.e., just numbers, the 'best' set shown in bold in right column) from an iterative process involving repeated chiral bi-sedenion multiplication. If noise injection at non-propagating ("off-shell") parameter $x_9$ is introduced then have non-zero components $\{\Delta x_0, \delta x_1, \delta x_2, \ldots \delta x_7, \delta x_8, \boldsymbol{\delta x_9}, \delta x_{16}\}$. The table lists the off-shell parameter, its asymptotic rc value, the full-width at half maximum (FWHM) of the peak (FWHM=2.335$\sigma$), and the inverse of the variance (taken as the best set of 'letters' available at this time).