PHYSICAL REVIEW D, VOLUME 60, 124017

Physical distinction among alternative vacuum states in flat spacetime geometries

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(Received 16 July 1998; revised manuscript received 7 July 1999; published 24 November 1999)

Even in flat spacetime, the states of a quantized field can be described via a variety of inequivalent Fock-space representations, associated with different congruences of inertial or noninertial observers. But it appears possible to distinguish among the possibilities on physical grounds: Field positive- and negative-frequency eigenfunctions might be required to be well defined and regular throughout the spacetime, so that the states can be attained by evolution from regular data in the remote past. This criterion distinguishes the familiar Minkowski-coordinate construction from that corresponding to the diverging congruence of observers whose world lines trace out a degenerate-Kasner subspace of Minkowski spacetime, for example. It also draws a physical distinction between the Minkowski-coordinate Fock-space states and those associated with a congruence of uniformly accelerated observers (Rindler observers); the latter states cannot be represented as any combinations of the former. This analysis of alternative descriptions of a quantized field may extend to more general classes of observers, and to more general spacetime geometries as well. [S0556-2821(99)05822-1]

PACS number(s): 04.62.+v, 03.70.+k

I. INTRODUCTION

The Fock space of states central to ordinary flat-spacetime quantum field theory is determined by the global timelike Killing vector field present in Minkowski spacetime. In more general spacetimes there are inequivalent vacua leading to different Fock space representations for a quantum field. Indeed, such Fock representations appear even in flat Minkowski spacetime: they are associated with congruences of observers other than those whose world lines delineate the familiar Minkowski coordinates. The problems of defining vacuum states, Fock spaces, and particles have occupied researchers in curved-spacetime quantum field theory since the inception of the subject [1–5]. The significance of these problems even in flat spacetime was highlighted in the seminal work of Fulling [2], Hawking [3], Davies [4], and Unruh [5], since elaborated upon by many authors [6].

In flat spacetime, however, it appears possible to select *on physical grounds* a particular vacuum state and Fock space from among those which can be defined by various congruences of observers. This might be done by requiring field positive- and negative-frequency eigenfunctions to be well defined and regular (e.g., at least C^2) throughout the spacetime, so that the states can be said to evolve from regular data in the distant past. Note that this criterion is applied to the positive-frequency eigenfunctions obtained directly from the defining Lie-derivative eigenvalue equation. Superpositions of these fundamental positive-frequency modes are still regarded as positive-frequency functions, though they may not satisfy the regularity criterion (just as for Fourier series). As we show in detail in this work, such a regularity criterion suffices, for example, to distinguish the field-theory description constructed by observers whose world lines trace out a degenerate-Kasner [7,8] subspace of Minkowski spacetime from that obtained in Minkowski coordinates. It also clarifies the physical distinction between the vacuum-particle definitions associated with uniformly accelerated (Rindler) observers and those of inertial observers.

There are three common approaches to the construction of Fock-space representations for quantized-field states in the general case: the C^* algebra, complex structure, and ''positive- and negative-frequency splitting'' methods [9]. The C^* algebra and complex structure approaches afford great generality, but require more elaborate formalism than is called for here. Instead we shall rely on the more familiar positive- and negative-frequency splitting method. This gives rise to a Fock-space representation by using a timelike vector field and a foliation of space-time. On each folium positive- and negative-frequency field modes are defined via a Lie-derivative eigenvalue equation, the Lie-derivative being taken along the given vector field and evaluated at the folium. (This is a well-known and straightforward generalization of the Minkowski-coordinate time derivative.) The field decomposed into these modes on any given folium defines creation and annihilation operators with which a vacuum and Fock-space states are constructed as usual. The Fock spaces obtained by implementing this procedure on different hypersurfaces of a given foliation in general need not be unitarily

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equivalent. However, if the congruence of observers moves along the integral curves of a Killing vector field, then there is a single vacuum and Fock space associated with the hypersurfaces of the foliation. As the well-known example of Fulling shows [2], in a region of spacetime with two linearly independent Killing vector fields, the vacua and Fock spaces associated with these two Killing vector fields may be unitarily inequivalent. The Fock spaces thus obtained are intimately connected with the vector field and foliation, i.e., with the congruence of observers involved. The inequivalence of these representations for different congruences of observers constitutes an ambiguity in vacuum and particle definitions.

The observer congruences associated with the degenerate-Kasner [7,8] and the Rindler [10,11] subspaces of Minkowski spacetime are convenient illustrations of the use of our physical criterion to clarify the ambiguity. The former consists of all observers moving with uniform speeds in a single spatial direction, their trajectories filling the forward light cones of all points of the plane perpendicular to their motion. (These Kasner observers do not move along integral curves of a Killing vector field.) The latter consists of the well-known congruence of observers moving with constant acceleration in a single spatial direction; their trajectories fill the region (''Rindler wedges'') outside the light cones of all points of a plane perpendicular to their direction of motion. The Killing-vector-field integral curves which constitute the Rindler-observer trajectories make them a favorite choice for modeling noninertial observers.

Our analysis shows that while the Kasner observers can generate a Fock-space representation of a quantized field with vacuum different from the familiar Minkowski vacuum, our physical criterion will single out the Minkowski vacuum in the degenerate-Kasner subspace, just as it does for a field theory on the full Minkowski spacetime. Likewise we find a clear distinction between the Fulling vacuum state obtained via positive- and negative-frequency splitting using the Rindler congruence, and the ordinary Minkowski vacuum state. The Fulling vacuum and associated states are based upon positive-frequency field functions which are singular on the Rindler horizon bifurcation event, and have discontinuous first derivatives at the horizon—all regular events in the complete spacetime. Hence our criterion selects the inertial Fock space as the appropriate physical framework for the field theory, even when accelerated or more general observers are involved. (Of course the Fulling vacuum and associated states remain of physical significance, e.g., in spacetimes with boundaries imposed by accelerating mirrors [12].) Furthermore, our criterion might prove useful in more general spacetimes, where no timelike Killing vector field exists. (Related criteria are considered in Refs. [13,14].)

The spacetime geometries, coordinate systems, and quantum-field-theory formalism we use here are outlined in Sec. II. Our analysis of field theory in the degenerate-Kasner subspace is described in Sec. III; Rindler-space theory is examined in Sec. IV. In Sec. V we summarize our findings and discuss their application to more general observers and more general spacetime geometries.

Our general notation and sign conventions here follow those of Misner, Thorne, and Wheeler [15]. We use units with $\hbar=c=1$ throughout.

II. GEOMETRIC AND FIELD-THEORETIC FORMALISM

A. The spacetimes

The geometries we consider are degenerate Kasner spacetime and Rindler spacetime, both subspaces of flat Minkowski spacetime. The Kasner solutions of the vacuum Einstein equations [7,8] correspond to universes which expand along two spatial directions and contract along a third. They have metric

$$ds^2=-d\tau^2+\tau^{2p_1}d\chi^2+\tau^{2p_2}d\psi^2+\tau^{2p_3}d\zeta^2 \qquad (2.1)$$

with τ, χ, ψ, and $\zeta\in(-\infty,\infty)$, and are characterized by the three numbers p_1, p_2, and p_3. In order to satisfy the vacuum Einstein equations the exponents satisfy

$$p_1+p_2+p_3=1 \quad \text{and} \quad p_1^2+p_2^2+p_3^2=1. \qquad (2.2)$$

The degenerate Kasner spacetime is the solution with $p_1=1$ and $p_2=p_3=0$, henceforth labeled K_{100}. It is more easily treated in a different coordinate system, namely,

$$t=\frac{1}{g}\ln(g\tau), \quad x=\frac{1}{g}\chi, \quad y=\psi, \quad \text{and} \quad z=\zeta. \qquad (2.3)$$

The metric is then

$$ds^2=e^{2gt}(-dt^2+dx^2)+dy^2+dz^2. \qquad (2.4)$$

It is easily seen that K_{100} is a region of Minkowski spacetime (henceforth M^4). The K_{100} coordinates are related to the Minkowski coordinates by

$$X=g^{-1}e^{gt}\sinh(gx), \quad T=g^{-1}e^{gt}\cosh(gx),$$

$$Y=y, \quad \text{and} \quad Z=z, \qquad (2.5)$$

yielding the familiar metric

$$ds^2=-dT^2+dX^2+dY^2+dZ^2. \qquad (2.6)$$

However, the transformation only maps into the region of M^4 with $T>|X|$. If K_{100} is shown as a subsection of M^4 using the Penrose diagram representation, the difference between their Cauchy surfaces is seen to depend only on contributions at sections of future null infinity (see Fig. 1). The form of the coordinate relation also shows that surfaces of constant t in K_{100} provide a foliation by hyperbolic sheets in the upper wedge of M^4.

The regions of Minkowski spacetime complementing $T>|X|$ and $T<-|X|$ (''time-reversed'' K_{100}) are the Rindler wedges. Since K_{100} and Rindler spacetime (henceforth $\mathcal{R}$) have trivial coordinate relations with Minkowski spacetime in the Y-Z plane, those two dimensions are dropped in the remaining analysis. The Rindler wedges, in the X-T plane, are covered by coordinates $\eta\in(-\infty,+\infty)$, $\xi\in(-\infty,+\infty)$,

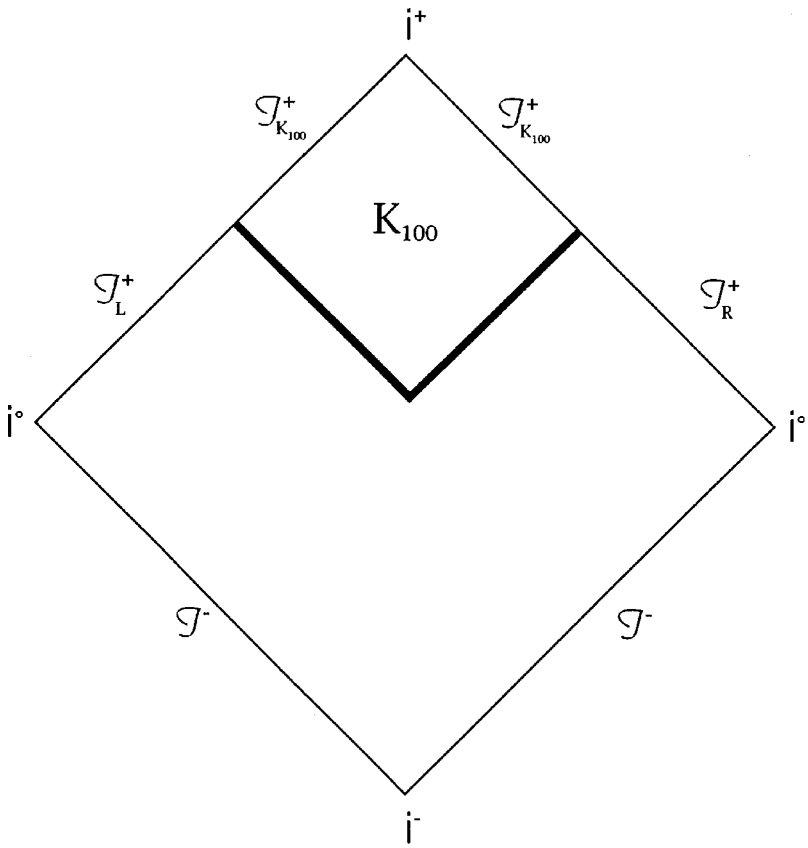

FIG. 1. The degenerate Kasner universe shown as a subsection of Minkowski spacetime. A Cauchy surface in K_{100} is a partial Cauchy surface in M^4, requiring information at the $\mathcal{I}^+_L$ and $\mathcal{I}^+_R$ portions of null infinity for completeness.

and a discrete coordinate $Q=\pm 1$ which distinguishes the right and left wedges. These are related to the Minkowski coordinates via

$$T=g^{-1}e^{g\xi}\sinh(g\eta)$$

$$X=Qg^{-1}e^{g\xi}\cosh(g\eta), \tag{2.7}$$

with g a constant (acceleration). The Rindler metric is thus

$$ds^2=e^{2g\xi}(-d\eta^2+d\xi^2), \tag{2.8}$$

in either wedge.

The Minkowski spacetime and its sections are maximally symmetric, i.e., they have ten Killing vector fields [16]. Restricted to the X-T plane, the three independent Killing vector fields are the time translation

$$\frac{\partial}{\partial T}=\begin{cases} e^{-gt}\left(\cosh(gx)\dfrac{\partial}{\partial t}-\sinh(gx)\dfrac{\partial}{\partial x}\right) & \text{in } K_{100}, \\ e^{-g\xi}\left(\cosh(g\eta)\dfrac{\partial}{\partial \eta}-\sinh(g\eta)\dfrac{\partial}{\partial \xi}\right) & \text{in } \mathcal{R}, \end{cases} \tag{2.9}$$

the spatial translation

$$\frac{\partial}{\partial X}=\begin{cases} e^{-gt}\left(\cosh(gx)\dfrac{\partial}{\partial x}-\sinh(gx)\dfrac{\partial}{\partial t}\right) & \text{in } K_{100}, \\ Qe^{-g\xi}\left(\cosh(g\eta)\dfrac{\partial}{\partial \xi}-\sinh(g\eta)\dfrac{\partial}{\partial \eta}\right) & \text{in } \mathcal{R}, \end{cases} \tag{2.10}$$

and

$$X\frac{\partial}{\partial T}+T\frac{\partial}{\partial X}=\begin{cases} \dfrac{\partial}{\partial x} & \text{in } K_{100}, \\ \dfrac{\partial}{\partial \eta} & \text{in } \mathcal{R}, \end{cases} \tag{2.11}$$

a boost in the X direction.

B. Quantum field theory

We consider a real scalar field of mass m with Lagrangian density

$$\mathcal{L}(x)=-\frac{\sqrt{-g}}{2}[g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi+(m^2+\xi R)\varphi^2], \tag{2.12}$$

where R is the Ricci scalar and ξ the curvature-coupling constant (not to be confused with the Rindler spatial coordinate ξ; here the distinction will always be clear from context). Flat spacetime has $R=0$; the field equation obtained from this Lagrangian density is the familiar Klein-Gordon equation $(\Box - m^2)\varphi = 0$. A complete set of orthonormal mode solutions, denoted $u_k(x)$, are used to describe φ:

$$\varphi(x) = \sum_k [a_k u_k + a_k^\dagger u_k^*]. \qquad (2.13)$$

Canonical quantization of the theory is implemented by imposing the usual commutation relations, $[a_k, a_{k'}^\dagger] = \delta_{kk'}$, etc. The vacuum is defined by $a_k|0\rangle = 0$ for all k. A different complete set of solutions, distinguished by overbars, likewise yields

$$\varphi(x) = \sum_q [\bar{a}_q \bar{u}_q + \bar{a}_q^\dagger \bar{u}_q^*], \qquad (2.14)$$

for which the new vacuum is defined by $\bar{a}_q|\bar{0}\rangle = 0$.

III. VACUA IN DEGENERATE KASNER SPACETIME

Various timelike vector fields might be used to define positive-frequency modes, hence a vacuum, for a quantized field in the K_{100} subspace. Symmetry considerations immediately distinguish two choices—the Killing and conformal-Killing vector fields. To choose between these, or among less symmetric possibilities, we seek a physical criterion for a preferred vacuum state. Here we propose the requirement that field positive-frequency eigenfunctions be well defined and regular (at least C^2) throughout the complete spacetime, so that the resulting states can be obtained via evolution from regular data in the remote past.

The conformal Killing vector field consists of the four-velocities of observers moving on world lines of constant (χ, ψ, ζ) in the Kasner coordinate system. This vector field is not Killing, owing to the τ dependence of the metric (2.1), and it

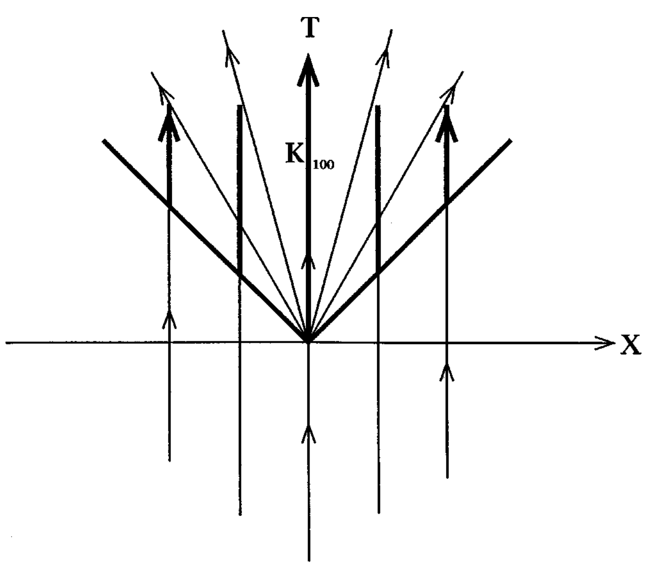

FIG. 2. Congruences of the inertial Killing vector field in K_{100}, and their extension to the remainder of Minkowski spacetime. Congruences of the conformal Killing vector field are also shown converging on the origin.

has a caustic at the Kasner-coordinate origin. With this choice of velocity field the vacua obtained via splitting of positive-negative frequencies will differ at different times τ, as shown in detail below. The positive-frequency modes associated with the conformal Killing vector field are irregular at the K_{100} boundaries $(X = \pm T)$, where they display oscillatory singular behavior, as shown in Ref. [5]. Thus, the conformal Killing vector field and associated vacuum states will not satisfy our criterion.

In constrast, an actual Killing vector field and corresponding Fock-space construction do satisfy our criterion, and yield the same vacuum on all folia. The timelike Killing vector fields on K_{100} can be described as the Minkowski time-translation vector field $\partial/\partial T$, and the vector fields obtained from this by uniform Lorentz boosts. These vector fields extend smoothly from the K_{100} subspace to the entire M^4 spacetime, as do the positive-frequency modes they define (see Fig. 2).

Field quantization in K_{100} begins with separation of the Klein-Gordon equation in the (t,x,y,z) Kasner coordinates (2.3). The separated mode functions can be expressed in terms of Hankel functions

$$u_{\mathbf{k}} = [c_{\mathbf{k}}^{(2)} H_{iK}^{(2)}(s) + c_{\mathbf{k}}^{(1)} H_{iK}^{(1)}(s)] e^{i\mathbf{k}\cdot\mathbf{x}} \qquad (3.1a)$$

or Bessel functions

$$u_{\mathbf{k}} = [d_{\mathbf{k}}^{(1)} J_{iK}(s) + d_{\mathbf{k}}^{(2)} J_{-iK}(s)] e^{i\mathbf{k}\cdot\mathbf{x}}. \qquad (3.1b)$$

(The latter case, for massless fields, requires additional information pertaining to the $k_x = 0$ modes.) The argument of the Hankel or Bessel functions is $s(t) = g^{-1}(k_y^2 + k_z^2 + m^2)^{1/2} e^{gt}$ and the order is $\pm iK = \pm i|k_x|/g$. A prescription for choosing positive-frequency modes would single out a particular combination of the Hankel or Bessel functions. A corresponding combination of the c's or d's is then associated with the annihilation operators (2.14) of the quantized field theory, thus defining a vacuum state.

The choice of positive-frequency modes based on the four-velocities of the Kasner-coordinate comoving observers is implemented by requiring the positive-frequency normal-mode functions $u_{\mathbf{j}}$ to be eigenfunctions of the Lie derivative with respect to proper time along the observers' world lines. That is, the positive-frequency eigenfunctions are required to be at least C^2 (i.e., ''regular'') functions satisfying

$$\mathcal{L}_{\partial/\partial\tau} u_{\mathbf{j}} = -i\Omega_{\mathbf{j}} u_{\mathbf{j}}. \qquad (3.2)$$

(Superpositions of these fundamental positive-frequency eigenmodes might still be regarded as positive-frequency functions, though they might not satisfy the regularity criterion.) Since the Lie derivative $\mathcal{L}_{\partial/\partial\tau}$ does not commute with the Klein-Gordon operator, the positive-frequency functions selected by imposing this condition at different times are different linear combinations of Hankel or Bessel functions, giving rise to a distinct vacuum state on each constant-τ hypersurface and consequent ''particle production.'' Such behavior has been studied in Refs. [17], where positive-frequency functions were chosen on the basis of asymptotic behavior at early and late Kasner times, thus defining distinct ''in'' and ''out'' vacuum states. But as noted above, neither this choice of positive-frequency modes nor the conformal-Killing vector field itself satisfies our criterion of extensibility throughout the spacetime, suggesting that these vacua may be classified as ''unphysical.''

Alternatively, the Minkowski time-translation Killing vector field can be used. Then the positive-frequency Lie-derivative eigenvalue equation

$$\mathcal{L}_{\partial/\partial T}\overline{u}_{\mathbf{l}}^{(+)}=-i\omega_{\mathbf{l}}\overline{u}_{\mathbf{l}}^{(+)} \tag{3.3}$$

picks out functions of the form

$$\begin{aligned}\overline{u}_{\mathbf{l}}^{(+)}&=e^{-i\omega_{\mathbf{l}}T}\lambda_{\mathbf{l}}(X,Y,Z)\\&=\exp[-i\omega_{\mathbf{l}}g^{-1}e^{gt}\cosh(gx)]\\&\quad\times\lambda_{\mathbf{l}}[g^{-1}e^{gt}\sinh(gx),y,z],\end{aligned} \tag{3.4}$$

where $\overline{u}_{\mathbf{l}}^{(+)}$ satisfies the Klein-Gordon equation $(\Box-m^2)\overline{u}_{\mathbf{l}}^{(+)}=0$ with $\omega_{\mathbf{l}}=(l_X^2+l_Y^2+l_Z^2+m^2)^{1/2}$ and $\lambda_{\mathbf{l}}(X,Y,Z)=\exp(il_XX+il_YY+il_ZZ)$. These positive-frequency normal modes are the familiar plane waves in Minkowski spacetime. They are regular throughout and clearly satisfy our criterion. A suitable combination of these mode functions can be written in terms of the Hankel or Bessel functions of Eq. (3.1). Such a combination remains positive-frequency; it serves to define the same vacuum state as modes (3.4), but is expressed in Kasner coordinates. Examination of the integral representation

$$H_{iK}^{(2)}(s)=\frac{-e^{-\pi K/2}}{i\pi}\int_{-\infty}^{\infty}e^{-is\cosh\rho-iK\rho}d\rho \tag{3.5}$$

reveals that the $H^{(2)}$ mode functions are the indicated superposition of positive-frequency functions [18], as defined by Eq. (3.3). Recalling $s=g^{-1}(k_y^2+k_z^2+m^2)^{1/2}e^{gt}$, and substituting $\rho=gx-\rho'$, we can rewrite this as

$$\begin{aligned}H_{iK}^{(2)}(s)e^{i\mathbf{k}\cdot\mathbf{x}}=&\frac{-e^{-\pi K/2}}{i\pi}\int_{-\infty}^{\infty}d\rho'\\&\times\exp\{-i[(k_z^2+k_y^2+m^2)^{1/2}\cosh\rho']\\&\times[g^{-1}e^{gt}\cosh(gx)]\}\\&\times\exp\{i[(k_z^2+k_y^2+m^2)^{1/2}\sinh\rho']\\&\times[g^{-1}e^{gt}\sinh(gx)]+iK\rho'\}e^{ik_yy+ik_zz}\\=&\frac{e^{-\pi K/2}}{i\pi}\int_{-\infty}^{\infty}d\rho'\exp\{-i\omega_{\mathbf{l}}(\rho')T+il_X(\rho')X\\&+iK\rho'+il_YY+il_ZZ\},\end{aligned} \tag{3.6}$$

with $\omega_{\mathbf{l}}$ and l_X parametrized by ρ': $\omega_{\mathbf{l}}(\rho')=(k_z^2+k_y^2+m^2)^{1/2}\cosh(\rho')$, $l_X(\rho')=(k_y^2+k_z^2+m^2)^{1/2}\sinh(\rho')$, $l_Y=k_y$, and $l_Z=k_z$. This choice of positive frequency serves to define the same vacuum state as the solutions of Eq. (3.3) at all times, with no ''particle production.'' Although the $H_{iK}^{(2)}(s)$ functions have oscillatory singular behavior (hence no derivative) on the boundaries ($X=\pm T$) of the K_{100} region, the fundamental definition of positive-frequency eigenfunctions in this instance derives from the normal modes (3.4), which are regular throughout M^4. The criterion is thus satisfied, and the grouping $H_{iK}^{(2)}(s)$ represents a positive-frequency solution of the Klein-Gordon equation separated in the Kasner coordinates.

Any other choice of timelike Killing vector field in the Lie-derivative eigenvalue equation in the K_{100} region gives the same result. In the $x-t$ plane these Killing vector fields are given by

$$\begin{aligned}\partial/\partial T'&=\cosh\vartheta\,\partial/\partial T+\sinh\vartheta\,\partial/\partial X\\&=e^{-gt}\left(\cosh(gx-\vartheta)\frac{\partial}{\partial t}-\sinh(gx-\vartheta)\frac{\partial}{\partial x}\right),\end{aligned} \tag{3.7}$$

with $\vartheta\in(-\infty,\infty)$ the usual boost parameter. Hence boosting from one vector field ξ to another is equivalent to replacing the Kasner coordinate x by $x-\vartheta/g$. Such a transformation multiplies each function $H_{iK}^{(2)}(s)e^{i\mathbf{k}\cdot\mathbf{x}}$ by a constant phase; consequently, these functions remain a basis of positive-frequency solutions. The vacuum state thus selected is the same for all values of the parameter ϑ. This choice of vacuum on K_{100} spacetime shares the Lorentz (boost) invariance of the usual inertial vacuum defined on the full M^4 spacetime.

What does this preferred choice of vacuum state in K_{100}, as determined by our criterion, imply about the state of the quantum field in the full Minkowski spacetime? A Cauchy surface in K_{100} can be extended to a Cauchy surface for M^4 by appending suitable portions of $\mathcal{I}^+$ (i.e., $\mathcal{I}_R^+$ and $\mathcal{I}_L^+$ in Fig. 1). Since for a massive field no physical quantity has support on $\mathcal{I}^+$, all inner products, quantum states, etc., defined on the M^4 Cauchy surface will be identical to those on the Cauchy surface in the K_{100} subspace. For massless fields, owing to possible contributions from $\mathcal{I}^+$, all the Cauchy data in M^4 is not determined by the Cauchy data in K_{100}. However, the set of states corresponding to the additional Cauchy data is of measure zero relative to the set of all states in M^4. In comparison with two-dimensional Kasner spacetime, on the other hand, the additional set of states is not of measure zero.

IV. VACUA AND FOCK SPACES FOR ACCELERATED OBSERVERS

The Rindler subspace traced out by a congruence of uniformly accelerated observers provides an example in which different *Killing* vectors used in the definition of positive frequency give rise to inequivalent descriptions of the quantized field. The Rindler observers' four-velocities are associated with a timelike Killing vector field in the $\mathcal{R}$ region (defined in Sec. II), one which is distinct from that associated with inertial (Minkowski) observers but which cannot be extended smoothly to the entire M^4 spacetime and remain both timelike and Killing. There is of course an enormous literature on quantum field theory in Rindler spacetime [2,4,5,6,11]; several authors have pointed out distinctions between the Rindler- and Minkowski-spacetime Fock spaces and vacuum states [2,4,5], which emerge despite apparent formal symmetry between the two treatments. Here those distinctions can be interpreted in terms of the nonextensibility of the Rindler construction, i.e., the absence of a smooth timelike and Killing extension of the Rindler timelike Killing field, and the oscillatory singular behavior of the Rindler modes at the horizons.

Our regularity criterion selects the inertial-observer positive-frequency modes over those identified via the Rindler-observer congruence. The timelike Killing vector field $\partial/\partial\eta$ tangent to the world lines of the accelerated observers (in two spacetime dimensions) yields positive-frequency modes

$$u_{R,k}=(4\pi\omega)^{-1/2}e^{ik\xi-i\omega\eta} \tag{4.1a}$$

in the right wedge R (with zero support in wedge L), and

$$u_{L,k}=(4\pi\omega)^{-1/2}e^{ik\xi+i\omega\eta} \tag{4.1b}$$

in the left wedge L (with zero support in wedge R), where $\omega=|k|$ is imposed by the Klein-Gordon equation for the simple case of a massless field [11]. These modes have oscillatory singularities on the Rindler horizons; they cannot be smoothly extended beyond the Rindler wedges and remain positive-frequency mode functions associated with a timelike vector field. Analyticity properties (in the Rindler coordinates) [5,11] imply that the combinations

$$\begin{aligned}\bar{u}_k^{(1)}&=[2\sinh(\pi\omega/g)]^{-1/2}(e^{-\pi\omega/2g}u^*_{R,-k}+e^{\pi\omega/2g}u_{L,k}),\\ \bar{u}_k^{(2)}&=[2\sinh(\pi\omega/g)]^{-1/2}(e^{\pi\omega/2g}u_{R,k}+e^{-\pi\omega/2g}u^*_{L,-k}),\end{aligned} \tag{4.2}$$

are positive-frequency functions in the inertial (Minkowski) description, i.e., these are superpositions of Minkowski-spacetime positive-frequency solutions. As in the Kasner example, the regularity of those basic solutions over the whole of M^4 satisfies the criterion.

The requirement that states be obtainable from regular data in the remote past differentiates the Minkowski vacuum and Fock-space structure from the ostensibly similar Rindler-spacetime constructions. Physical consequences arise from the oscillatory singular behavior of the Rindler positive-frequency mode functions, and the inextensibility of the Rindler congruence as a timelike Killing vector field, at the Rindler horizons. Quantities such as stress-energy expectation values in Fulling-Rindler Fock-space states are divergent there [19]. But these are ordinary points of the M^4 spacetime. Hence regular evolution from regular data in the distant past could not be expected to put a field into these states. The Minkowski vacuum and Fock-space states do not suffer from such drawbacks, and may thus be distinguished as the physical Hilbert space of the quantized field. Hence, for example, in treating the interaction between a field and an observer on an *arbitrary* world line, it would be appropriate to represent the state of the field using the Fock space constructed with a congruence of inertial (Minkowski) observers.

V. SUMMARY

These results show that the familiar Minkowski-coordinate Fock-space representation of the states of a quantized field stands out on simple physical grounds from the diverse array of alternative representations which can be constructed even in flat spacetime. The Minkowski-coordinate timelike Killing vector field (time translation) used to define positive- and negative-frequency normal modes extends over the entire M^4 manifold without singularity; the resulting normal-mode functions are regular on the entire spacetime; renormalized stress-energy expectation values in the associated Fock-space states likewise exhibit no singularities anywhere on M^4. Hence these states of the quantized field can be obtained by evolution from regular data in the remote past. Alternative constructions of vacuum states and Fock spaces may fail to satisfy one or more of these criteria.

The degenerate-Kasner geometry traced out by the conformal-Killing observers, moving with all possible velocities perpendicular to a fixed spacelike plane, admits a variety of choices of vacuum state and positive-frequency modes for a quantized field. Inequivalent choices can be made at different ''times,'' giving rise to ''particle production [17].'' But via the regularity criterion described here, a unique choice can be made corresponding to the Minkowski-coordinate construction carried out on the full spacetime. Then no particles are produced, and the theory is—at least for massive fields—exactly equivalent to the familiar Minkowski-spacetime theory.

The Fulling-Rindler vacuum and Fock space associated with a congruence of uniformly accelerated observers are a well-known example of an alternative representation of a quantized field. But this construction violates our regularity criterion, and the resulting field states exhibit a variety of pathologies [2,4,5]. In particular, since these states give rise to stress-energy divergences on the light cones which are the horizons of the Rindler coordinates, they cannot arise via regular evolution from the remote past. Hence it is the Minkowski Fock-space states—the construction of which satisfies our criterion—which should represent the physical states of the field.

In general spacetimes, lacking the extensive isometry structure of M^4, choosing an appropriate vacuum state and Fock space from the infinity of possibilities is problematic. But the criteria considered here of extensibility over the entire spacetime, of nonsingularity on the entire spacetime (except, of course, at actual physical singularities of the geometry), of evolution from the remote past (or initial singularity), may prove useful in these more general circumstances as well.

ACKNOWLEDGMENTS

This research was supported in part by the National Science Foundation under Grant No. PHY-9105935. S.W.-H. acknowledges his indebtedness to Nicolas J. Papastamatiou (now deceased) for many enlightening discussions.

[1] L. Parker, Ph.D. thesis, Harvard University, 1966; Phys. Rev. **183**, 1057 (1969).
[2] S. A. Fulling, Phys. Rev. D **7**, 2850 (1973).
[3] S. W. Hawking, Nature (London) **248**, 30 (1974).
[4] P. C. W. Davies, J. Phys. A **8**, 609 (1975).
[5] W. G. Unruh, Phys. Rev. D **14**, 870 (1976).
[6] A. Ashtekar and A. Magnon, Proc. R. Soc. London **A346**, 375 (1975); S. A. Fulling, Gen. Relativ. Gravit. **10**, 807 (1979); U. H. Gerlach, Phys. Rev. D **38**, 514 (1988); **38**, 522 (1988); extensive literature cited in Ref. [11].
[7] E. Kasner, Am. J. Math. **43**, 217 (1921).
[8] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973), pp. 800–816.
[9] A. Ashtekar and A. Magnon, Proc. R. Soc. London **A346**, 375 (1975); A. Ashtekar and A. Magnon, Gen. Relativ. Gravit. **12**, 205 (1980); T. Dray, J. Renn, and D. Salisbury, Lett. Math. Phys. **7**, 143 (1983); A.-H. Najmi and A. C. Ottewill, Phys. Rev. D **30**, 1733 (1984); A.-H. Najmi and A. C. Ottewill, Gen. Relativ. Gravit. **17**, 573 (1985); P. Chmielowski, report given at 1992 Midwest Relativity Conference, Univ. of Illinois at Champaign-Urbana.
[10] W. Rindler, Am. J. Phys. **34**, 1174 (1966).
[11] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1982), pp. 109–117.
[12] G. T. Moore, J. Math. Phys. **9**, 2679 (1970); S. A. Fulling and P. C. W. Davies, Proc. R. Soc. London **A348**, 393 (1976).
[13] Ch. Charach and L. Parker, Phys. Rev. D **24**, 3023 (1981).
[14] I. H. Redmount, Phys. Rev. D **60**, 104004 (1999).
[15] Misner, Thorne, and Wheeler [8], frontispiece.
[16] S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972), pp. 375-404.
[17] T. Padmanabhan, Phys. Rev. Lett. **64**, 2471 (1990); see also C. W. Misner, Phys. Rev. D **8**, 3271 (1973); B. K. Berger, *ibid.* **12**, 368 (1975).
[18] B. L. Hu, S. A. Fulling, and L. Parker, Phys. Rev. D **8**, 2377 (1973).
[19] P. Candelas and D. Deutsch, Proc. R. Soc. London **A354**, 79 (1977); **A362**, 251 (1978); N. D. Birrell and P. C. W. Davies, Ref. [11], p. 186; R. Parentani, Class. Quantum Grav. **10**, 1409 (1993).